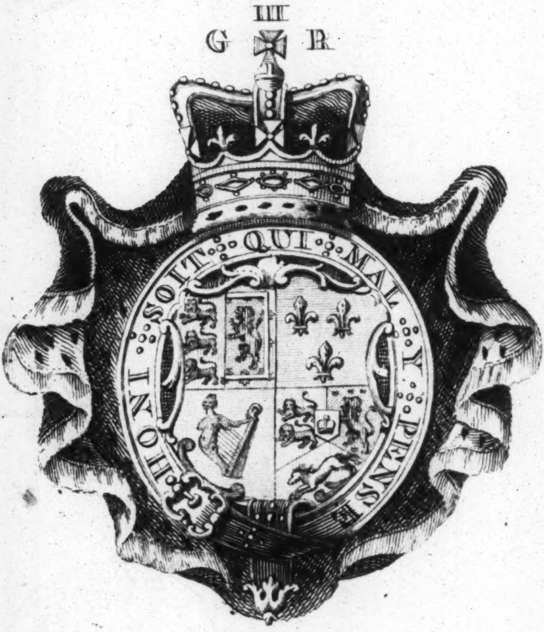
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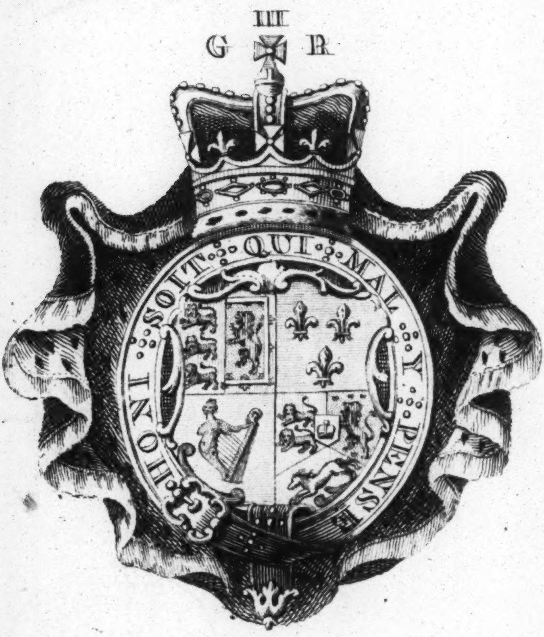
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A
COMPENDIOUS COURSE
OF
Practical Mathematicks.

Particularly Adapted to the
USE of the GENTLEMEN
OF THE
ARMY and *NAVY*.

In THREE VOLUMES.

For the Moſt Part tranſlated from the Tracts publiſh'd
in *French* by *P. Hoſte*, Profeſſor of Methematicks in
the *Royal Academy* of *Thoulon*,

By *WILLIAM WEBSTER*.
The SECOND EDITION,
Corrected, with a few Remarks, by *Ellis Webſter*.

VOL. II. Containing,

I. Elements of EUCLID.
II. TRIGONOMETRY.
III. Practical GEOMETRY.
IV. MECHANICKS.
V. FORTIFICATION.
VI. GUNNERY.

LONDON:
Printed for D. BROWNE, without *Temple-Bar*, and
C. HITCH, and L. HAWES, in *Pater-Noſter-Row*,
M.DC.LI

BR. MUS.

To His Grace

JOHN,

Duke of ARGYLE and GREENWICH, Maſter-General of His Majeſty's Ordnance, &c.

May it pleaſe Your GRACE,

THE collection of Tracts which form this Volume, containing ſuch parts of Mathematical knowledge as are properly the ſtudy of a Soldier, ſeems to claim a kind of privilege to Your Graces's protection, as a General, and Maſter of the Ordnance: Nor can there be a greater inducement to the Gentlemen of the Army to regard theſe neceſſary Qualifications, than the countenance of Your Grace, whoſe great Example they will be juſtly ambitious to follow.

ARMS have been Your early exerciſe, and ARTS Your conſtant ſtudy: So that the eminent Figure Your Grace has long made, both in the Field and Senate, is a topick upon which even a cold Imagination might with eaſe expatiate: But tho' juſt Praiſe is undoubtedly a grateful Incenſe, yet there is a certain delicacy in the Offering, which I dare not attempt.

I therefore humbly beg pardon for the preſumption of this Addreſs, and crave leave, with the moſt profound reſpect, to ſubſcribe myſelf,

May it pleaſe Your GRACE.

Your Grace's moſt Obedient,

And moſt Humble Servant,

William Webſter.

THE

Author's PREFACE.

BEING obliged by my Station to ſtudy the propereſt means of inſtructing the Marine Officers, in ſuch parts of the Mathematicks as are neceſſary for their Qualification, I thought the moſt effectual method would be to lay before them a Collection, wherein they might both ſee what they had to learn, without troubling themſelves with any other Books, and might with eaſe reviſe what they had already been inſtructed in.

I conſidered, that this Collection ought to be as ſhort as poſſible; that the Perſons for whom it is intended might not be diſcouraged with the length of the work; and I may reaſonably hope that every Gentleman concerned will endeavour to render himſelf duly qnalified, when he obſerves, that the whole courſe of his Mathematical Studies are comprehended in three little Volumes.

In the entrance upon each Subject I have let nothing paſs without clear explication and demonſtration; but in the progreſs, I have choſe to leave the Reader the pleaſure of diſcovering by himſelf ſome of the moſt eaſy concluſions; of which, however, I have given him proper intimations

I have not been ſcrupulouſly exact in rendering my method of demonſtration perfectly uniform, and in deducing my proofs immediately from firſt Principles, or the next preceding Propoſitions; but have rather made it my chief endeavour to avoid every thing that might embarraſs, or diſcourage my Readers: And when ſeveral demonſtrations have offered themſelves in proof of the ſame Propoſition, I have have always choſe that which appeared to me moſt intelligible and ſhort, rather than that which perhaps might be thought more exact.

THE

PREFACE.

THIS ſhort Courſe of practical Mathematicks, which I have ventured to tranſlate from the French, *for the benefit of the* Engliſh *Reader, will, I hope, by its brevity and perſpicuity, recommend itſelf to ſuch Gentlemen as think ſome knowledge in theſe uſeful Sciences a laudable Accompliſhment. How neceſſary this Study is, eſpecially in the Articles which make up this Collection, to thoſe who would rightly quallify themſelves either for the* Army *or* Navy, *is too evident to be inſiſted on; but how conducive the preſent Performance may be to that uſeful purpoſe, is not for me to determine; as a Tranſlator, I am become a party in the Deſign. I ſhall therefore preſume no farther, than humbly to offer a word or two in behalf of my Author, with reſpect to the ſeveral parts of his Work,* *and to ſubjoin, as I go on, a ſhort Account of the liberties I have taken in rendring it into* Engliſh.

He begins with a ſhort extract from the firſt ſix, and the eleventh and twelfth books of Euclid, *wherein, I think, he has ſhewn great judgment, in collecting all that is neceſſary for his purpoſe, without burdening his Reader with any propoſitions foreign to his deſign; ſo that I think it may be juſtly ſaid, there is nothing extant of the like kind, at once ſo compleat and ſo conciſe: Nor do I judge it unworthy of remark, that the propoſitions ſtand numbered as they follow in the books of* Euclid. *With regard to the tranſlation of this part, I have one thing to advertiſe the Reader,* viz. *That whereas the Author had referr'd his extract from the fourth book of* Euclid, *to his tract upon* Practical Geometry, *I thought it better to give all the Elements together ſince references might as well be made to that, Part as the reſt.*

The ſecond ſubject of our Author is Arithmetick, *upon which having myſelf wrote more fully, I entirely omitted his Tract; yet I hope it is not ill ſupply'd with my* Arithmetick in Epitome, *which may ſerve as the firſt Volume of this Courſe.*

Trigonometry *is his third tract, in which he has pretty copiouſly handled the* Doctrine of Triangles, *both plain and ſpherical, and has therein alſo given a ſhort, but clear, explication of the conſtruction and uſe of Logarithms.*

Practical Geometry *is his next ſubject, wherein he ſuccinctly treats of the manner of taking Heights and Diſtances, of the method of making Surveys, and of the meaſurement of Surfaces and Solids. This part in the Original contains likewiſe ſome extracts from the fourth Book of* Euclid, *which, as I before obſerv'd, in the Tranſlation, I have rang'd in their proper order among the Elements, with ſome additions. He has likewiſe therein given the deſcription of ſome Inſtruments, for which I muſt refer the Reader to the Appendix I have added at the end of the laſt Volume.*

The Uſe of the Globes *is the next tract in the order of our Author; but being deſirous to compriſe all that more immediately regarded the Land-Service in one Volume, I have placed this Tract in the beginning of the laſt Volume, as more adapted to Maritime Affairs. All I ſhall ſay of this Treatiſe is, that it is very ſhort, and yet I think ſufficiently intelligible.*

Next follows a tract upon Mechanicks, *the ſhorteſt and moſt intelligible I have ever met with. There is one Article in this Treatiſe I have taken the liberty to alter (tho' indeed I rather believe it to be an error of the Printer, than a miſtake of the Author) ſince the change of but one word,* viz. autant *into* moins, *that is,* as much *into* leſs, *makes the obſervation juſt, and the ſenſe conſiſtent with the Corollary following. See* Pag. 160. Fig. 21.

Fortification *and* Gunnery *are the two remaining tracts, which make up one Volume in this Tranſla-*

tion, in which both thoſe ſubjects are ſuccinctly handled. In the one, the young Officer will ſee the maxims and methods of Military Architecture, with a proper explanation of Terms; and in the other, the Mathematical Principles applicable to Artillery, with juſt and neceſſary obſervations on the laws of motion with reſpect to Projectiles.

Theſe being ſubjects much out of my own ſphere, I therein conſulted proper Judges: And muſt particularly own my obligations to Captain Richards *of his Majeſty's Train of Artillery, for his kind aſſiſtance with reſpect to the latter.*

The two tracts on Navigation *contain'd in the other volume, are (eſpecially the latter) uncommon Eſſays upon the ſubject: In the one is ſhewn the conſtruction and uſe of Inſtruments, Charts, and Tables; with the application of* Trigonometry *to Sailing in the calculation of the Courſe, Diſtance, and Departure; alſo the neceſſary* Addenda *of finding the* Prime, Epact, Golden Number, Moon's Age, *and time of* High Water *in any Port,* &c. *In the other, which concerns the working of a Ship (a ſubject not treated on that I know of by any* Engliſh *Author) is conſider'd and explain'd, from the eſtabliſh'd laws of Motion, and the form and ſtructure of a Veſſel, the ſeveral motions a Ship is capable of receiving, and the proper methods of giving them; which neceſſarily led the Author into the conſideration of the force and efficacy of the ſeveral Sails, according to their different ſituation; as alſo the power of the Rudder in changing the line of Direction. I have only to obſerve under this head, that as ſome of the practices he delivers differ from the methods uſed by the*

the Engliſh, *I have ſet you down the* Engliſh *manner of performance by way of Notes at the bottom of the page; which likewiſe contain whatever other obſervations or remarks have occurr'd throughout the whole Performance.*

In this part of the Work, I have been alſo careful to get proper aſiſtance; and am eſpecially obliged to my good friend Captain Wilde, *without whoſe Help it had been impoſſible for me to have underſtood the* French *terms, or to have expreſs'd them with any tolerable propriety in* Engliſh: *But my Friend being abſent when the laſt hand was ſet to the Work, I muſt ſtill beſpaek the indulgence of the* Engliſh *Sailor, if he finds my expreſſions ſometimes deviate from the true Sea dialect.*

I have now done with our Author. As to the two Articles I have added, they are, Firſt, *An alphabetical explanation of ſuch Sea terms and phraſes as occur in the Work, which will not only be uſeful to ſuch Gentlemen as are deſirous to look into ſo curious a Theory, though not concern'd with the Sea; but even neceſſary to thoſe who undertake the ſtudy, with a view to their future practice.* Secondly, *An* Appendix, *containing the deſcription and uſe of a compleat ſet of Pocket Inſtruments, wherein the little* Apparatus, *ſo neceſſary to every young Mathematician, is ſufficiently explain'd; and the ſeveral lines laid down upon the* Plain Scale, Gunter Scale, *and* Sector, *are conſidered, both with regard to their conſtruction and application.*

Having thus laid before the Reader a ſummary account of the Work, I ſhall not farther take up his time with apologiſiing for the performance, I have done my beſt endeavour, both in the Tranſlation and the additional Articles; I therefore ſubmit the whole to the candid peruſal of the Publick, well knowing that books of Science can only make their way by their Merit.

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BR. MUS.

E L E-

ELEMENTS

OF

EUCLID.

BOOK I.

DEFINITIONS.

1. MATHEMATICKS are certain Sciences, by which we learn to compare things one with another, in regard to their extenſion, or to determine whether they are equal, or in what proportion one thing is greater, or leſs than another.

2. *Magnitude* therefore is that idea ariſing in our minds, from the compariſon of one thing with another, in regard to extenſion.

3. *Extenſion* is to be conſidered in three different reſpects, which we call Dimenſions, *viz.* 1. Length. 2. Breadth. 3. Thickneſs, or Depth.

4. A *Solid*, or a *Body*, is a magnitude conſidered under all theſe three dimenſions; as if the tonnage of a Ship is demanded, the Ship is conſidered as a *Solid*; and the queſtion requires that we examine its length, its breadth, and its depth; for the longer, the broader, and the deeper it is, the greater will be its tonnage.

5. A *Superficies*, or *Surface*, is a magnitude comprehended under but two dimenſions, *viz.* length, and breadth; as when we meaſure a field, we take it for a *Surface*, which the longer, and broader it is, the more acres it contains, without any regard to the depth, or thickneſs of the earth.

6. A *Line* is a magnitude in which we conſider only length; as when it is aſk'd, how far one place is diſtant from another, we conſider the ſpace between the two places as a line, which, the longer it is, the farther diſtant are the places; but the breadth, or thickneſs of the ſpace, makes no difference in the diſtance of the places.

7. A *Point* is that in which we conſider neither length, breadth, nor thickneſs, as in the foregoing Inſtance, the two places are taken for two Points; for it is not at all neceſſary to know the length, breadth, or thickneſs of the two places, to determine the diſtance between them.

8. The two extreams of a Line are Points: a Line being only a continuation, or ſucceſſion of Points. So Lines are the boundaries of a Superficies, which is nothing but a web of Lines. In like manner alſo, Superficies are the boundaries of a ſolid Body, which is compoſed of a heap, or multitude of Superficies, connected, or laid one upon another.

9. Two Lines, or two Surfaces are ſaid to be equal, when the one being laid upon the other, perfectly covers it. In like manner, two Solids are accounted equal, when one being ſuppoſed to penetrate the other, is perfectly contain'd within the ſame limits.

10. A right Line, is the ſhorteſt diſtance between two Points: thus the Line A B is a right Line, as, on the contrary, the Lines C D are curve, or crooked. *Fig.* 1.

11. A plain Surface is that upon which a right Line may be every way applied: Or a Plain is a web of right Lines. Thus the Surface A B C D is plain, but the Surface E F G H curve. The hollow ſide of a curve Surface is called concave, the raiſed ſide convex. *Fig.* 2.

12. A Line is perpendicular to another, when it has no inclination on either ſide, as A B upon C D. *Fig.* 3.

13. The Line A B is parallel to the Line C D, if all the perpendicular Lines between them are of equal length. *Fig.* 4.

Fig. 5. 14. A Circle is a plain, whereof all the extremities are equally diſtant from one certain point, as A B C. The Circumference of a Circle, is the curve line which incompaſſes it, as A B C. The Center is the point E, in the middle. A right line drawn from the Center to the Circumference, is call'd a Radius, as B E; and if, in the ſame manner, a right line be drawn quite croſs the Circle, paſſing thro' the Center, it is called a Diameter, as A E D: laſtly, an Arch is any part of the Circumference, as B C.

Fig. 6. 15. A plain Angle, is the diſtance or opening of two lines, which touch in one point, not making one line; thus the opening of the lines B A, B C, makes the Angle A B C.

16. An Angle is the greater, the wider the lines are apart: But as the length of the lines makes no alteration in their diſtance; ſo it makes no difference in the largeneſs, or ſmallneſs of the Angle.

Fig. 7. 17. The meaſure of an Angle, as A B C, is found by deſcribing a Circle round the point B, which Circle being divided into 360 equal parts, called Degrees; the number of ſuch parts contained in the Arch A C, is the quantity of the Angle in Degrees.

Fig. 8. 18. A *right* Angle is that which is exactly meaſured by an Arch of 90 Degrees, or the ſourth part of the circumference of a Circle, a Circle, as A B C. An *obtuſe* Angle is greater than a right Angle, as E B C, and an *acute* Angle leſs, as E B D.

19. A rectilinear Angle is ſorm'd by two right lines; a curvilinear by two curves; and a mix'd Angle by one line right, *Fig.* 6. and one curve.

20. A Triangle is a plain Surface bounded by three lines, as A, B, C. 1. If the three lines are equal, the Triangle is equilateral; *Fig.* 9. 2. If two of them are equal, it is an Iſoſceles Triangle; 3. But if the three lines are all unequal, the Triangle is call'd a Scalenum.

21. A Parallelogram is a plain bounded with four lines, whereof the oppoſite ones are parallel, as A B C D. *Fig.* 10. There are four ſorts of Parallelograms: 1. The true Square, which has all its ſides and angles equal. 2. The long Square, which has its angles equal, but not its ſides. 3. The Rhombus, which has its ſides equal, but not its angles. 4. The Rhomboides, which has neither its ſides, nor angles equal.

22. We call a Truth which we demonſtrate, a *Propoſition*. Of which there are ſeveral ſorts: 1. *Theorems*, which are truths purely ſpeculative. 2. *Problems*, wherein ſomething is propoſed to be done. 3. *Lemmas*, which are truths laid down only for the demonſtration of others. 4. *Corollaries*, which are the conſequences drawn from a Propoſition demonſtrated.

23. *Axioms* are plain truths known to all the World; ſome of the principal of which are as follow:

1. The whole is greater than one of its parts.

2. Two things are equal to each other, when each one is equal to a third.

3. If two equal things are equally augmented or diminiſhed, they will remain equal; but if they are augmented or diminiſhed unequally, they will become unequal.

N. B. *The Propoſitions are numbered as they ſtand in* Euclid.

PROPOSITION I.

To make an equilateral Triangle upon the Line AB. From the point A with *Fig.* 11. the Compaſſes open'd to the extent AB, deſcribe the Arch BC; and from the point B, with the ſame extent, deſcribe the arch AC, which ſhall cut the arch BC, in the point C; then drawing the lines AC, and BC, each of them ſhall be equal to the line AB, (by *Def.* 14.) and to one another (by *Ax.* 2.) conſequently the Triangle ABC, will be equilateral, (according to *Def.* 20.)

PROPOSITION IV.

If the line AB of one Triangle, be equal to the line ED of another Triangle, and *Fig.* 12. alſo the line AC equal to the line EF. If, farther, the angle A of the one, be equal to the angle E of the other, the two Triangles are equal in every reſpect.

Demonſtration. If the line E D, of the Triangle E D F, be laid upon the line A B, of the Triangle A B C, it will perfectly cover it, (by *Def.* 9.) and the line E F will fall upon the line A C; ſince then the angles A and E are equal, they will likewiſe exactly cover each other; ſo that the point F, will be found upon the point C, in the ſame manner as the point D, upon the point B; conſequently the line D F, muſt alſo cover the line B C, and the whole Triangle E D F, exactly cover the Triangle A B C. Therefore it is equal in every reſpect, (by *Def.* 9.)

PROP. V.

If the lines A B, AC, of the Triangle A B C, are equal, the angles B and C ſhall be alſo equal. Make another Triangle D E F, exactly equal to the Triangle A B C, ſo that the angle A, may be equal to the Angle D, and the lines A B, and A C, equal to the lines D E, D F. *Fig.* 13.

Demonſtr. If the angle D, be ſo laid upon the angle A, that the line D E falls upon the line A B, the Triangle D E F will perfectly cover the Triangle A B C, and the angle E will be found equal to the angle B; but if the line D E be laid upon the line A C, then will the angle E be found equal to the angle C; the angles therefore B and C being each of them equal to the angle E, are conſequently equal to one another, (by *Ax.* 2.)

PROP. VI.

If the angles B, and C, of the Triangle A B C are equal; the lines A B, A C are alſo *Fig.* 13. equal. Suppoſe, as before, D E F, equal in every reſpect to A B C.

Demonſtration. If the Triangle D E F, be transferr'd upon the Triangle A B C, by laying E F, upon B C, the one will perfectly cover the other; and if the point E, falls upon the point B, the line D E, will alſo be found equal to the line A B; but if the point E, be laid upon the point C, then will the line D E, be found equal to the line A C, and conſequently the lines A B, A C, being each equal to the line D E, muſt alſo (by *Ax.* 2.) be equal one to the other.

PROP. VIII.

If the lines A B, B C, of the Triangle A B C, are equal to the lines D E, E F, of the *Fig.* 14. triangle D E F; it is evident that the Baſe A C, cannot be equal to the Baſe D F, unleſs the angle B, be equal to the angle E: conſequently (by *Prop.* 4.) the three lines of the triangle A B C, cannot be equal to the three lines of the triangle D E F, without the two triangles are equal in every reſpect.

PROP. IX.

To divide an Angle, as B A C into two equal parts: Making the point A, a center, *Fig.* 15. deſcribe the Arch B C, and upon the line B C, (by *Prop.* 1.) make an equilateral triangle B D C, ſo ſhall the line A D, drawn from the angular point to the oppoſite interſection, divide the angle B A C into two equal parts; that is to ſay the angle B A D, ſhall be equal to the angle C A D.

Demonſtration. The triangle A B D, has all its ſides equal to the ſides of the triangle A C D; conſequently (by *Prop.* 4.) the angle B A D, muſt be equal to the angle C A D.

PROP. X.

To divide the line A B, into two equal parts: Make upon it (by *Prop.* 1.) the equilateral triangles B D A, B C A, then draw the line C D, which ſhall ſo divide the line A B in the point E, that the parts E B, and E A, ſhall be equal.

Fig. 16.

Demonſtrat. The triangles A D E, B D E have the ſide D E common, and the ſide B D is equal to the ſide A D, (by *Def.* 14.) and farther, the angle A D E, is equal to the angle B D E, (by *Prop.* 9.) Therefore is the triangle A D E, equal in every reſpect to the triangle B D E, (by *Prop.* 4.) and the baſe A E, equal to the baſe B E.

PROP. XI.

To raiſe from the Point A, a line perpendicular to the line B C: Make A B, A C equal, and from the points B and C, as centers, with the ſame opening of the Compaſſes deſcribe two arches cutting each other in the point D, and the line D A ſhall be a perpendicular; that is to ſay, the angles D A B,

Fig. 17.

D A C, ſhall be equal (by *Prop.* 8.) becauſe all the ſides of the triangle D A B, will be equal to all the ſides of the triangle D A C.

Prop. XII.

To let fall a Perpendicular upon a line B C, from a point given without it A. On *Fig.* 18. the point A deſcribe an arch, which ſhall cut the line in the points B, and C, then making the equilateral triangle B E C, (by *Prop.* 1.) the line A E ſhall be perpendicular to the line B C.

Demonſtrat. Since the triangles A B D, and A C D, have the ſide A D common, and A B equal to A C, and the angle B A D being alſo equal to the angle C A D, (by *Prop.* 9.) they are equal in every reſpect, (by *Prop.* 4.) and the angle D equal on each ſide; therefore is the line A D, perpendicular to the line B C, (by *Def.* 12.)

Prop. XIII.

The line D C falling upon the line A B, makes two angles *viz.* D C A, D C B, equal *Fig.* 19. to two right ones: for if from the center C a circle be deſcribed, the ſemicircle A D B will exactly meaſure them, (by *Def.* 18.)

Prop XIV.

If the two angles (in the foregoing Figure) *viz.* D C A, D C B, are equal to two right ones, the two lines A C, B C, muſt conſequently make but one line, which ſhall be the diameter of the circle A D B.

Prop. XV.

When two lines, as A B, C D, cut each other as in the point E, they form four angles, of which the oppoſite ones are equal, as B E D, C E A, becauſe either of them contain what is wanting in the angle B E C, to make two right ones, (by *Prop.* 13.) *Fig.* 20.

COROLLARY.

If the line E F, cuts the parallels A B, C D, the angle E H B, ſhall be equal to the angles C G F; becauſe the two parallels may be taken for one line, and then the two angles are oppoſite, as before, (by the preceding.) Farther, becauſe the angle A H G is equal to the angle E H B, as the angle H G D to the angle C G F, being oppoſite; therefore the four angles E H B, A H G, H G D, C G F, ſhall all be equal, the angles C G F, E H B, are called outward alternates; the angles A H G, H G D inward alternates; the angles E H B, H G D alternatively oppoſed. Theſe three ſorts of angles are then equal, when the two lines thus croſs'd by a third are parallel: and when they are equal, the two lines thus croſs'd by a third are parallels. *Fig.* 21.

Prop. XVI. and XXXII.

If any ſide, as B C, of the triangle A B C, be produced, or drawn out, it will make the exterior angle A C F, equal to both the interior and oppoſite angle A, B, *Fig.* 22.

Demonſtration. Thro' the point C, let the line C D be drawn, parallel to the line A B; which will divide the outward angle into two parts, of which the firſt, *viz.* A C D, will be equal to the angle A, and the ſecond, *viz* D C F, equal to the angle B, (by the preceding.)

COROLLARIES.

1. The three angles of a Triangle are equal but to two right angles: for the outward angle A C F with the adjoining angle A C B, are equal but to two right ones, (by *Prop.* 13.) therefore the angles A, B, with the ſame angles A C B, can be but equal to two right angles.

2. When therefore a triangle has one angle right, or obtuſe, both the other angles muſt be acute.

3. If two angles of one triangle are equal to two angles of another triangle, their third angles are alſo equal.

PROP. XVIII.

If the ſide A B of the triangle A B C is ſhorter than the ſide A C, the angle C, oppoſite to the ſide A B, ſhall be leſs than the angle B, oppoſite to the ſide A C. *Fig.* 23.

Take A D, equal to the ſide A B, and draw B D.

Demonſtration. The ſides A B, A D, of the triangle A B D, being equal, the angles A B D, A D B, ſhall be equal, (by *Prop.* 5.) but the angle A D B, being an outward angle, is greater than the angle C. (by the preceding) therefore the angle A B D will be greater than the angle C, and conſequently the angle A B C will be ſo much greater than the angle C.

PROP.

PROP. XIX.

If the angle B, is greater than the angle C, the ſide A C, ſhall be greater than the ſide A B. *Fig.* 23.

Demonſtrat. If the ſide A C, was not greater than the ſide A B, either it would be equal, and the angle B, would be equal to the angle C, (by *Prop.* 5.) or it would be leſs, and the angle C would be greater than the angle B, (by the preceding) both of which will be contrary to the ſuppoſition.

PROP. XX, XXI.

The two ſides A C, C B, of a triangle, taken together, are longer than the third A B, which being a right line, is the ſhorteſt diſtance from the point A, to the point B. *Fig.* 24. In the ſame manner the lines A D, D B, are longer than the lines A C, C B, becauſe they are further diſtant from the right line.

PROP. XXII.

To make a triangle of three given lines A B, C D, E F: Firſt lay down a right line equal in extent to the line A B, then *Fig.* 25. open the compaſſes to the extent C D, and with one Foot in the point B, deſcribe an arch; then taking the extent E F, between your compaſſes, with one foot in A, make another arch to cut the former in the point G; ſo ſhall the triangle A G B, (by *Def.* 14.) be compoſed of the line A B, and the lines A G, B G, equal to the given lines A B, C D and E F.

PROP.

Prop. XXIII.

To make on the point A, of the line A B, an angle equal to the angle D E F: From *Fig.* 26. the point E, as a center, deſcribe an arch D F, and from the point A, with the ſame extent of the compaſſes, deſcribe another arch B G; then opening the compaſſes to the diſtance D F, from the point B, as a center, make an arch cutting the arch B G, in the point H, and the angle B A H, ſhall be equal to the angle D E F; the two Triangles being (by *Prop.* 8.) equal in every reſpect.

Prop. XXIV.

If the ſides D E, and D F, of one triangle, are of the ſame length with the ſides C A *Fig.* 27. and A B, of another; when thoſe ſides are farther diſtant than theſe, that is, if the angle D, is greater than the angle A, the baſe E F, will alſo be greater than the baſe B C, which is ſelf-evident.

Prop. XXV.

If the ſide A B, of the triangle A B D, is equal to the ſide C F, of the triangle C E F; *Fig.* 28. if alſo the angle A, be equal to the angle C, and the angle B to the angle F, the two triangles ſhall be equal in every reſpect.

Demonſtration. If the triangle C E F, be laid upon the triangle A B D, in ſuch manner that the ſide CF, falls exactly upon the ſide A B, then will the angle C agree with the angle A, and the angle F with the angle B; conſequently the line C E, will fall upon the line A D, and the line F E, on the line B D, and the triangle C F E, perfectly cover the triangle A B D.

PROP. XXXI.

To draw thro' the point C, a line parallel to the line A B: Draw the line C D, and on the point C, make the angle D C E, equal to the angle C D A, (by *Prop.* 23.) and the line E C, ſhall be parallel to the line A B, (by *Cor.* to *Prop.* 15.) *Fig.* 29.

PROP. XXXIII.

If the lines A B, D C, are equal, and parallel; the lines A D, B C, ſhall be alſo equal, and parallel: Draw A C. *Fig.* 30.

Demonſtrat. The Triangles A B C, A D C, are equal in every reſpect, (by *Prop.* 4.) becauſe the ſide A C, is common to both, and the ſide A B, of the one, is ſuppoſed equal to the ſide C D of the other; and farther, the alternate inward angles B A C, D C A, are equal, (by *Cor.* to *Prop.* 15.) Therefore the baſe B C, muſt be equal to the baſe A D; and the angle A C B, equal to the angle C A D; and conſequently B C, will be parallel to A D.

PROP. XXXIV.

If A B is parallel to D C, and A D parallel to B C, the oppoſite ſides, and angles of the parallelogram A B C D, ſhall be equal. *Fig.* 30.

Demonſtration. The triangles A B C, A D C, having the baſe A C common, and the inward alternate angles B A C, D C A, C A D, A C B, equal, muſt be equal in every reſpect, (by *Prop.* 25.) therefore A B, is equal to D C; BC, equal to A D; and the angle B, equal to the angle D.

PROP. XXXV.

If the Parallelograms A B C D, E F C D, have the ſame Baſe C D, and are between *Fig.* 31. the ſame parallels C D, A F, they ſhall themſelves be equal.

Demonſt. All the ſides of the triangle A C E, being equal to the ſides of the triangle B D F, (by the preceding) the two triangles are equal, (by *Prop.* 8.) Therefore if from each, the triangle B G E, which is common to both, be taken away, the ſpace 1, which remains of the firſt, will be equal to the ſpace 2, the remainder of the ſecond; and conſequently the ſpace 1, with the triangle C D G, or the parallelogram A B C D, will be equal to the ſpace 2, with the ſame triangle C D G, or to the parallelogram E F C D.

PROP. XXXVI.

If the parallelograms A B C D, E F G H, which are between the ſame parallels A G, *Fig.* 32. B H, have their baſes B C, E G, equal; they will each of them be equal to the parallelogram B C G E, (by the preceeding) and conſequently one to another.

PROP. XXXVII.

If the triangles A B C, C E G, are between the ſame Parallels B F, A G; and the baſe A C of the one, equal to the baſe C G of the other, they are equal; becauſe the one is the half of the parallelogram A B D C, and the other is the half of the parallelogram C E F G, (by *Prop.* 34.) which is equal to the parallelogram A B D C, (by the preceding.) *Fig.* 33.

PROP. XLI.

If the Triangle A B E, has the ſame Baſe A B as the parallelogram A B D C, and ſtands between the ſame parallels A B, C E, it ſhall be the half of the parallelogram A B C D; in the ſame manner as the triangle A B C, which (by the preceding,) is equal to it. *Fig.* 34.

PROP. XLVII.

If the angle A of the triangle A F C is right, the ſquare of the ſide F C, which is oppoſite to it, ſhall be equal to the ſquare of the ſide A C, and the ſquare of the ſide A F. *Fig.* 35.

Suppoſe the three ſquares to be each made upon its proper ſide; the lines C A, G A, will then become one line, as will alſo the lines F A, H A, (by *Prop.* 14.) draw F D, and A B. I ſay, that the line A E, being drawn parallel to the line B C, ſhall divide the ſquare F B, into two rectangular parallelograms; of which the one

C E, ſhall be equal to the ſquare of the ſide A C; and the other F E, equal to the ſquare of the ſide A F.

Demonſt. the triangle F C D, having the ſame baſe C D, as the ſquare D A, and being between the ſame parallels F A, C D, will be juſt equal to its half, (by *Prop.* 41.) in the ſame manner the triangle A B C, will be half the rectangle C E. but the triangle F C D, is equal to the triangle A B C, (by *Prop.* 4.) becauſe the ſide C B is equal to the ſide C F, the ſide C A, to the ſide C D, and the angles A C B, F C D, are alſo equal, being each compoſed of one right, and the angle F C A. Therefore the half of the ſquare A D, is equal to the half of the rectangle C E, and conſequently the whole ſquare A D, will be equal to the whole rectangle C E.

In the ſame manner it may be ſhewn, that the ſquare F G, is equal to the rectangle F E.

BOOK II.

DEFINITIONS.

1. A *Rectangle*, is a Parallelogram, which has all its Angles right.

2. The Value, or Content of a Rectangle, is the number of ſquares which it contains: Therefore to find the content, *Fig.* 1. we multiply the length by the breadth.

3. A Rectangle is ſaid to be contain'd under two lines, one of which expreſſes its length, the other its breadth. Thus *Fig.* 1. the rectangle D, is contain'd under the lines A C, B C.

4. The *Diameter* or *Diagonal* of a rectangle, is a line drawn from one *Fig.* 2. angle to another which is oppoſite, as C B.

Prop. I. and II.

If the ſide A B, of a rectangle A B C H, be divided into a certain *Fig.* 3. number of parts, as ſuppoſe three, and upon each part be form'd a rectangle having the ſame height A C, with the rectangle A B C H: the three rectangles ſo made, will be equal to the rectangle A B C H. For of theſe rectangles A F, D G, E H, is compoſed the rectangle A B C H.

PROP. III.

Fig. 4. If the line A B, be divided in the point C, and the perpendicular A D, be equal to one of thoſe parts, as A C; the rectangle A B F D, will be equal to the ſquare of the part A C, to wit, A D E C; and the rectangle C E F B, which has the part A C for its breadth, and the other part C B for its length.

LEMMA.

Fig. 5. If thro' the point I, of the diameter of a ſquare D B, be drawn two lines parallel to the ſides B A, D A; there will be form'd four rectangles, of which the two that are croſs'd with the diagonal will be ſquares, to wit, G, and H.

Demonſt. All the angles of the Parallelogram G, are right, (by 34. 1.) farther all its ſides are equal; for in the right angled, and iſoſceles triangle D B A, the equal angles B, and D, being (by 5. 1.) equal but to one right, are each of them half right angles. So in the right angled triangle B I C, the angles I and B, being equal but to one right, the angle I, will be half a right angle, as the angle B; and conſequently the ſides B C, C I, will be equal (by 5. 1.) as alſo their oppoſites M I, M B; and the rectangle G, will be a ſquare.

PROP. IV.

Fig. 5.

If the line A B, be divided in the point C, the whole ſquare of the line A B, will be equal to the ſquare of the part A C, and the ſquare of the part B C, and two rectangles whoſe baſe ſhall be A C, and height B C; that is to ſay, the ſquares G, and H, and the rectangles L, K, (by *Lemma* preceding.)

PROP. V.

Fig. 6.

If the line A B, is divided equally in the point C, and unequally in the point D, the rectangle of the unequal parts A D, D B, with the ſquare of the line C D, will be equal to the ſquare of the part C B.

Demonſt. The line D G, or A H, is equal to the line D B, (by *Lemma* preceding) and the line B E, is equal to the line C A, by the ſuppoſition; therefore the rectangles D E, C H, are equal; and if to each be added the rectangle D F, then will the whole ſquare C E, be equal to the whole figure K D A, which is compoſed of the rectangle of the unequal parts D H, and of the ſquare of the line C D, or F K.

PROP. VI.

Fig. 7.

If the line A B, be divided equally in the point C, and to it be annex'd the line B D; the ſquare of the line C D, will be equal to the rectangle K A, whoſe length is the whole line A D, and breadth the annex'd part B D, with the ſquare H G, whoſe ſide is the line C B.

Demonſt. The line B D, or H K, is equal t the line F A, (by *Lemma* preceding) and the li A C, or C B, is equal to the line H I ; therefo the rectangle K I, is equal to the rectangle C F adding then to both, the figure G C D K H I, th rectangle K A, with the ſquare G H, will be equ to the ſquare C E.

P R O P. XI.

Fig. 8.

If a ſquare be form'd upon the line A B, a one of its ſides, as A C, be equally d vided in the point D : If, farther, th line D G, be drawn equal to the li B D ; the ſquare A G H F, ſhall be equal to th rectangle F E.

Demonſt. Since the line A C, is equally div ded in the point D, and is lengthened by the li A G, the rectangle C H, with the ſquare of th line A D, will be equal to the ſquare of the li D G or D B, (by the preceding :) But the ſqua A E, with the ſquare of the line A D, is alſo equ to the ſquare of the line D B, (by 47. 1.) Ther fore the ſquare A E, is equal to the rectangle C H taking then away from both the rectangle C the rectangle F E, will be equal to the ſqua F G.

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BOOK III.

DEFINITIONS.

1. WHEN one line meets another without cutting it, we ſay it touches, or is a *Tangent*: Thus the line A B touches, or is a *Tangent* to the circumference of the circle C. *Fig.* 1.

2. The *Segment* of a circle is a part cut off with a cord line: or it is a figure bounded by a right line, and an arch of a circle; as A B C. *Fig.* 2.

3. A *Sector* is a part of a circle bounded by two radii, or ſemidiameters, and the included arch; as A B C. *Fig.* 3.

4. An angle is ſaid to be in a ſegment, when the right line of the ſegment is taken for the baſe, and the angular point touches the arch: Thus the angle A B C, is ſaid to be in the ſegment A B C. *Fig.* 4.

PROP.

PROP. I.

If from the middle of the line B
Fig. 5. be rais'd the perpendicular A F, it ſha
paſs thro' the center of the circle F B

Demonſt. If any Point E, not in the line A be the center of the circle F B C, the triangl BEA, CEA, will be equal in every reſpe (by 8. 1.) then would the line E A, be perpendicular to B C, and not F A, as was ſuppoſed.

PROP. II.

If from the center D, of a circl
Fig. 6. D B be drawn perpendicular upon A
it ſhall divide the line A C into tw
equal parts B A, B C.

Demonſt. In the triangles A B D, C B D, whic are rectangled at the point B, the angles C, and A being equal, (by 5. 1.) their complements B D A B D C, will be alſo equal; and becauſe B D i common to both triangles, and the ſide D equal to the ſide D C, the two triangles ſhall b equal in every reſpect, (by 4. 1.) and the baſe B A, B C equal.

PROP. VII.

If from the point A, in a circle, of which it i
not the center, ſeveral lines be drawn
Fig. 7. to the circumference; the line A B,
which paſſes thro' the center C, ſhal
be longer than any other as A D, &c.

Demonſt.

Demonſtrat. The line A C B is equal to the two ſides A C, C D, of the triangle A C D; therefore it is longer than the ſide A D, (by 20. 1.)

Again, A D ſhall be longer than A E, which is farther diſtant from the center.

Demonſt. The lines A C, C D, being of the ſame length as the lines A C, C E, and making the angle A C D greater than the angle A C E, will alſo make the baſe A D greater than the baſe A E, (by 24. 1.)

Laſtly, A F, which makes one line with A C B, ſhall be leſs than any other, as A E, *&c.*

Demonſt. F A, with A C, are equal but to C E; whereas A E, with A C, are longer than C E, (by 20. 1.)

PROP XVI.

If the line B C is perpendicular upon the end of the diameter A B, it will touch, or be a tangent to the circle. *Fig.* 8.

Demonſt. If it were to cut the circle, ſome of its points, as C, would enter, or be within the circle; and the line A C, being oppoſite to the right angle B, would be leſs than the line A B, which is oppoſite to an acute angle C; which (by 18. 1.) cannot be.

LEMMA I.

If the line A B touch the circle, and the line A C cuts it; the angle B A C ſhall be meaſured by half the arch A C. Draw from the center D, the line D E, perpendicular to C A; ſo that it may equally divide the line A C, in the point G, and the arch A C, in the point E, (by 1. 3.) *Fig.* 9.

Demonſt.

Demonſt. The angle A D G, with the angle D A G, are equal but to one right, (by 16. 1.) and the angle B A C, with the ſame angle D A G, will be equal but to one right, (by 16. 1.) therefore the angle B A C, is equal to the angle A D G, which is meaſured by E A, half the arch A C.

The angle C A F, ſhall be alſo meaſured by half the arch C H A; becauſe (by 16. 1.) the conſequent angles B A C, C A F, are meaſured by half the whole circle.

LEMMA II.

The angle A B C, whoſe point touches the concave circumference of the circle, is meaſured by half the arch A C, upon which it reſts. Draw the tangent D B F.

Fig. 10.

Demonſtrat. The three angles D B A, A B C, C B F, are meaſured by half the circle, (by 13. 1.) the angle D B A is meaſured by half the arch B A; and the angle C B F by half the arch C B, (by *Lemma* preceding) therefore the angle A B C ſhall be meaſured by half the remainder A C.

PROP. XX.

The angle A C B, at the center, is double the angle A D B, at the circumference of a circle; ſince (by *Lemma* preceding) the latter is meaſured by the half of the arch A B, the whole of which arch is the content or meaſure of the former.

Fig. 11.

PROP. XXI.

All ſuch angles of the circumference as reſt upon the ſame arch are equal; each having for its meaſure (by *Lemma* preceding) half the arch upon which it reſts.

PROP. XXII.

If the four angles of the quadrilateral figure A B C D, fall into the circumference of a circle, the oppoſite angles A C, or B D, are equal to two right; becauſe the angle A is meaſured by half the arch B C D, and the angle C, by half the arch B A D.

Fig. 12.

PROP. XXV.

To find the center of a circle whoſe circumference ſhall paſs through three given points, A, C, E. Biſect the Lines A C, C E, in the points B, D, and draw the perpendiculars B G, D F, and the point H, where they cut each other, ſhall be the center of the circle; ſince (by 1. 3.) both the lines B G, and D F, muſt paſs through the center.

Fig. 13.

PROP. XXXI.

If an angle at the circumference A B C reſts upon the ſemicircle A C, it is a right angle; but the angle B D C, which reſts upon the arch B A C, greater than a ſemicircle, is an obtuſe angle; and the angle

Fig. 14.

C A B, reſting upon the arch B D C, leſs than a ſemicircle, is an acute angle; all which is evident from *Lemma* 2. preceding.

P R O P. XXXII.

Fig. 15.

If you draw the tangent B A C, and the line A D, to cut the circle in the point of contact, the angle C A D ſhall be equal to the angles of the alternate ſegment A F D, becauſe (by *Lemma* 2.) both are meaſured by half the arch A D. In the ſame manner the angle B A D ſhall be equal to all the angles of the ſegment A E D.

P R O P. XXXIII.

Fig. 16.

To deſcribe upon the line A B an arch whoſe half ſhall be the meaſure of the angle C. Make the angle B A D equal to the angle C, (by 23. 1.) then having drawn A F, perpendicular to A D, make the angle A B E equal to the angle B A F, and the line B E ſhall cut the line A F, in the point G, which ſhall be the center of a circle, of which the equal lines G A, G B, ſhall be radii, (by 6. 1.) the line A D a tangent, (by 16. 3.) and half the arch A B ſhall be the meaſure of the angle B A D, or the angle C, (by *Lemma* 2.)

P R O P. XXXIV.

Fig. 17.

To cut off ſuch an arch A C, of a circle, as that its half ſhall be the juſt meaſure of a given angle D, or, in other words, a circle being given, to cut from it a ſegment capable of a certain angle. Draw the tangent A B, and make the angle B A C, equal to the angle D.

Prop. XXXV, XXXVI

Shall ſerve as *Corollaries* to the 16th of the 6th Book.

BOOK IV.

DEFINITIONS.

1. A Rectilinear figure is ſaid to be inſcribed in a circle; or a circle is deſcribed about a rectilinear figure, when all the angles of the one fall into the circumference of the other. Thus the triangle A B C is inſcribed in the circle A B C D. *Fig.* 1.

2. A rectilinear figure is deſcribed about a circle; or a circle is inſcribed within a rectilinear figure, when all the ſides of the one touch the circumference of the other. Thus the triangle ABC is deſcribed about the circle D E F. *Fig.* 2.

3. A line is inſcribed in a circle, when both its extremes touch the circumference. Thus the line A B is inſcribed in the circle A B C. *Fig.* 3.

P R O P. I.

Fig. 4. To inſcribe in a circle A E B D a line B C, not exceeding its diameter; take between the feet of your compaſſes the length of the line B C, and ſetting one foot in one of the extremities of the diameter, deſcribe a circle cutting the circle AEBD in D, and E; then draw the line BD, or BE, which will be equal to BC, (by *Def*. 14. 1.)

PROP. II.

To inſcribe in a circle a triangle EGH, equiangular to another triangle ABC. Draw the tangent FD, and at the point of contact E make the angle DEH equal to the angle B, and the angle FEG equal to the angle C, (by 23. 1.) then draw the line GH, and the triangle EGH will be equiangular to the triangle ABC. *Fig*. 5.

Demonſt. The angle DEH is equal to the angle EGH of the alternate ſegment, (by 32. 3.) but the angle DEH is equal to the angle B, and conſequently the angles B and G are equal, (by *Ax*. 2.) by the ſame reaſon the angles C and H are alſo equal; then (by *Cor*. 2. of 16. or 32. of 1.) the angles A and GEH will be equal: therefore the triangles EGH and ABC are equiangular.

PROP. III.

To deſcribe a triangle LMN, about a circle GHK, equiangular to another triangle ABC. Produce one ſide of the given triangle, as BC, to D, and F; then making the angle GIH equal to the angle ABD and the angle HIK equal to the angle ACF, draw the tangents LGM, LKN, and NHM, through the points G, K, H, and they ſhall concur in angles equal to the angles given. *Fig*. 6.

Demonſt.

Demonſtrat. All the angles of the quadrilateral G I H M are equal to four right angles, becauſe it may be divided into two triangles. The angles I G M and I H M, which are made by the tangents, are right angles; therefore the angles M and I are equal to two right angles, as are alſo the angles A B C, and A B D: but the angle G I H, is equal to the angle A B D: therefore the angle M, will be equal to the angle A B C. By the ſame reaſon the angles N and A C B are equal; and therefore the triangles L M N and A B C are equiangular (by *Corol.* 2. of 16. or 32. 1.)

PROP. IV.

To inſcribe a circle E F G, in a triangle A B C.

Fig. 7. Biſect the angles B, and C, (by the 9. 1.) drawing the lines B D, and C D, which will concur in the point D; from which point D draw the perpendiculars D E, D F, and D G, which will be equal, (by *Def.* 14. 1.) ſo that a circle deſcribed from the center D, at the diſtance D E, will paſs through F and G.

Demonſtrat. The triangles D E B and D F B have the angles E and F equal, being both right; the angles D B E and D B F are alſo equal, and the ſide D B common: therefore the triangles will be equal in all reſpects, (by 25. 1.) and the ſides D E and D F equal. In the ſame manner may the ſides D F and D G be proved equal. A circle may then be drawn, which ſhall paſs through the points E, F, G; and becauſe the angles E, F, and G, are right angles, the ſides A B, A C, and B C, will touch the circle, which will conſequently be inſcribed in the triangle.

Prop. V.

To deſcribe a circle about a triangle. See *Prop.* 25. 3.

Prop. VI. VII.

To inſcribe a ſquare in a circle A C B D, or to deſcribe a ſquare about the ſaid circle. Having drawn the two diameters A B and C D, cutting each other perpendicularly at the center E; for the inſcribed ſquare, draw the lines A C, C B, B D, D A, and it is done: and for the circumſcribed ſquare, it is made by drawing the tangents F G, G H, H I, and I F, all which is evident. *Fig.* 8.

Prop. VIII.

To inſcribe a circle in a ſquare. Biſect the ſeveral ſides, and the point E, where the lines concur, will be the center. *v. Fig.* 8.

Prop. IX.

To deſcribe a circle about a ſquare. Draw the diagonals A B and C D, and the point E, where they cut each other, will be the center. *v. Fig.* 8.

PROP. X.

To deſcribe an iſoſceles triangle A B D, having the baſe angles B and D, each *Fig.* 9. double to the angle A. Divide the line A B, (by 11. 2.) ſo that the ſquare of A C may be equal to the rectangle under A B and B C; and from the center A, at the diſtance A B, deſcribe the arch B D, in which draw B D, equal to A C; then drawing the line D C, deſcribe a circle about the triangle A C D, (by *Prop.* 5.)

Demonſtration. Since the ſquare of A C, or B D, is equal to the rectangle of A B and B C; and the line B D touches the circle A C D, at the point D; therefore the angle B D C will be equal to the angle A, comprehended in the alternate ſegment C A D, (by 32 3.) Now the angle B C D, being an external angle in reſpect of the triangle A C D, is equal to the angles A, and C D A; therefore the angle B C D is equal to the angle B D A. Farther, the angle A D B is equal to the angle A B D, (by 5. 1.) therefore D C B and D B C are equal, and (by 6. 1.) the ſides B D and D C will be equal; and ſince B D is equal to A C, the ſides A C, and C D will be equal, (by *Ax.* 2.) and ſo likewiſe the angles A, and C D A; therefore the angle A D B, is double the angle A.

PROP. XI.

To inſcribe a regular Pentagon in a *Fig.* 10. circle. Deſcribe (by the preceding) an iſoſceles A B C, having each of its baſe angles B, and C, double the angle A. Then (by *Prop.* 2.) inſcribe within the circle the triangle D E F, equiangular to A B C; next divide the angles D E F, and D F E, each into two equal parts, and draw the lines E G, and F H; laſtly, join the lines D H, D G, G F, E H, and you will have deſcribed a regular Pentagon, that is to ſay, a Pentagon having equal ſides, and equal angles.

Demonſt. The angles D E G, G E F, D F H, and H F E, being the halves of the angles D E F, and D F E, each of which is double to the angle E D F, are equal to the angle E D F; and conſequently the five arches upon which they reſt, are equal, (by 21. 3.) ſecondly, the angles D G F, G F E, *&c.* having each three of thoſe equal arches for its baſe, will be alſo equal, (by *Ax.* 3.) therefore the ſides and angles of the pentagon D H E F G, are equal.

PROP. XII.

To deſcribe a pentagon about a circle. Inſcribe a regular pentagon A B C D E, in the circle, (by the preceding) and drawing tangents thro' the points A, B, C, D, E, (by the 16. 3.) you will have deſcribed a regular pentagon about the circle. *Fig.* 11.

PROP. XIII. XIV.

To inſcribe a circle in any regular Polygon; or to deſcribe a circle about a regular Polygon. Biſect any two ſides or two angles not oppoſite, and where the lines concur ſhall be the center.

PROP. XV.

To inſcribe a regular Hexagon in the circle ABCDEF; draw the diameter *Fig.* 12. AD, then with the compaſſes open'd to the radius of the circle, ſet one foot in the point D, and deſcribe the arch CE, then draw the diameters EGB, and CGF; and the lines AB, AF, FE, *&c.* ſhall form the Hexagon required.

Demonſtrat. 'Tis evident, that the triangles CDG, and DGE, are equilateral; therefore the angles CGD, DGE, and thoſe oppoſed to them at the top BGA, and AGF, are each of them the third part of two right angles; that is to ſay, contain 60 degrees each. Now all the angles that can be made about the ſame point, are equal but to four right angles, or 360 degrees: therefore taking away four times 60, that is, 240, from 360, there will remain 120 for BGC, and FGE, which therefore each contain 60 degrees; conſequently all the angles at the center being equal, all the arches and all the ſides will be equal; and every angle, as A, B, C, *&c.* will be compounded of two angles of 60 degrees each, that is 120 degrees, and therefore will be equal.

COROLLARY.

The ſide of a hexagon is equal to the ſemidiameter.

Prop. XVI.

To inſcribe a regular Pentedecagon in a circle.

Fig. 13.

Inſcribe in the circle an equilateral triangle, (by *Prop.* 2.) and a regular pentagon, (by *Prop.* 11.) ſo that the angles may meet at the point A, the line B F, will be the ſide of a Pentedecagon.

Demonſt. Since the line A B is the ſide of an equilateral triangle, the arch A E B will be the third part of the whole circle, that is, five fifteenths. But the arch A E, equal E F, being the fifth part, will contain three fifteenths, conſequently the arch B F, being one third of E F, will be one fifteenth.

As a Supplement to this Fourth Book of Euclid, *may be added this practical method, to divide a circle, as* A B C D, *into any number of equal parts, not exceeding ten.*

Fig. 14.

Deſcribe a circle, whoſe center let be E, and croſs it with two diameters A C, and B D, at right angles to each other; and ſet off from the point A, A O, and A Q, equal to B E the radius, and join O Q, ſo ſhall O Q be the third part of the circle; then join A B, which line A B will be the fourth part; upon L, with the diſtance L B, deſcribe the arch B M, and join B M, which line ſhall be the fiſth part; A E, or B E the radius, is the ſixth part; and L O, or L Q, the ſeventh part; biſect the angle A E B, and K A, will be the eighth part; divide the arch Q A O, into three parts at S, and join S Q, which will be the ninth part; laſtly, E M will be the tenth part.

BOOK

BOOK V.

DEFINITIONS.

1. A Part is a leſſer quantity compared with a greater; thus a rule of two foot is a part of one of five.

2. A Quantity which is exactly contain'd a certain number of times in another is call'd an aliquot part: thus a rule of two foot is an aliquot part of one of four foot; but is term'd an aliquant part of one of five foot. An aliquot part takes its particular name from the number of times it is contain'd in the quantity of which it is a part: as when it is contain'd twice, we call it a half; if thrice, a third; if four times, a fourth, *&c.*

3. The Ratio of one magnitude to another, is the relation that the one has to the other, with regard to the number of times it contains any of its aliquot parts: thus the Ratio of a rule of two foot with one of ſix, is the relation or reſpect it has to the rule of ſix foot, in that it once contains its third part, twice its ſixth, and four times its twelfth, *&c.*

4. There are two ſorts of Ratio's: 1. When one magnitude exactly contains a certain number of times an aliquot part of another, as a rule of two foot contains the third part of a rule of ſix foot exactly once, in ſuch caſe the ratio is call'd rational. 2. When one magnitude does not exactly contain any number of times an aliquot part of another; as it happens with numbers or quantities which are incommenſurable. For example, the diagonal of a ſquare is ſomething leſs than three times the half of its ſide, and ſomething more than five times the fourth part, *&c.* without being the exact meaſure of any part: in ſuch caſe the ratio is irrational.

5. In every Ratio there are two terms, the firſt of which is call'd the Antecedent, the other, the Conſequent: Thus in the Ratio of the magnitude A, to the manitude B, A is the Antecedent, and B the Conſequent; on the contrary, if the Ratio is expreſs'd, as B to A, then is B the Antecedent, and A the Conſequent.

6. If the antecedent exactly contains its conſequent a certain number of times, the Ratio is call'd multiple, and is ſaid to be double, triple, or quadruple, *&c.* according as the antecedent contains its conſequent twice, thrice, or four times, *&c.* but if, on the contrary, the antecedent is exactly contained a certain number of times in its conſequent, the Ratio is call'd ſub-multiple, and is expreſs'd according to the number of times, by the words ſub-double, ſub-triple, or ſub-quadruple, *&c.*

7. We expreſs the value of a rational Ratio by the figure ſhewing the number of times the antecedent contains its conſequent, or one of its aliquot parts; and theſe figures we call Exponents: thus 2 is the Exponent of double Ratio, and $\frac{1}{2}$ is the Exponent of ſub-double Ratio. In the ſame manner $\frac{3}{5}$ is the Exponent of a Ratio, wherein the antecedent contains three times the fifth part of its conſequent.

8. Two Ratio's, whether rational or irrational, are ſaid to be equal, when the antecedent of the one contains any aliquot part of its conſequent, ſo many times (whether exactly or not) as the antecedent of the other contains the ſame part of its conſequent: Thus not only two Ratio's double, or ſub-double, triple, or ſubtriple, are equal, and alſo all rational Ratio's which have the ſame Exponent; but even the irrational Ratio's A and B ſhall be equal, if the antecedent of the Ratio A contains the third part of its conſequent ſomething leſs than ſix times, and its ſixth part ſomething more then eleven times, *&c.* and the antecedent of the Ratio B, in the ſame manner contains ſomething leſs then ſix times the third part of its conſequent, and ſomething more then eleven times its ſixth part; and that the ſame may be ſaid of all the other parts both of the one, and the other conſequent.

9. Proportion is an equality of Ratio's: Thus we ſay, there is a proportion between the magnitudes or quantities $\left\{\begin{matrix}4 & 2 & 12 & 6\\ A, & B, & C, & D,\end{matrix}\right\}$ if the Ratio of

the quantity $\overset{4}{A}$ to the quantity $\overset{2}{B}$ is equal to the Ratio of the quantity $\overset{12}{C}$ to the quantity $\overset{6}{D}$, which is uſually expreſs'd in the following manner, A : B :: C : D, that is, as A is to B, ſo is C to D. Proportion requires at leaſt three Quantities; for there cannot be two Ratio's without at leaſt three Quantities.

PROP. VII.

If the quantities A and B are equal, they will have an equal ratio to any third quantity, as C; for the quantity A will contain every part of C, as many times as the quantity B will contain the ſame. In like manner the quantity C will have the ſame ratio to both the quantities A and B, becauſe it will contain any part of the one as many times as it will contain the ſame part of the other.

PROP. VIII.

If A is greater than B, it will have a greater ratio to C, becauſe it will oftner contain any of its parts. On the contrary, C will have a leſſer ratio to A than to B, becauſe the parts of A being greater than thoſe of B, they will be fewer times contain'd in C.

PROP. IX.

If the two quantities A and B have the ſame ratio to a third as C; or if C has the ſame ratio to A as to B; the quantities A and B are equal.

PROP.

PROP. X.

If A has a greater ratio to C than B; or if C has a leſſer ratio to A than to B; A muſt be greater than B.

Theſe two Propoſitions are demonſtrated as the foregoing

PROP. XI.

It is evident, that two ratio's are equal, if each is equal to a third.

PROP. XII.

If A : B : : C : D, then by conjunction A and C : B and D : : A : B.

Demon. If A contains twice the third part of B : C will contain twice the third part of D, and conſequently A and C will contain twice the third part of B and D.

PROP. XIII.

If A : B : : C : D, and A has a greater ratio to B, than E to F, it is evident, that C will have a greater ratio to D, than E to F.

PROP. XV.

It is alſo evident, that A is to B, as the half of A to the half of B, the fourth of A to the fourth of B, and ſo to the other aliquot parts of A, and of B.

PROP. XVI.

If A : B : : C : D, then by exchange, A : C : : B : D, ſuppoſing theſe four quantities of the ſame kind, that is all lines, or all ſuperficies, or all ſolids.

Demonſt. If A is $\frac{2}{3}$ of B, then will C alſo be $\frac{2}{3}$ of D; ſeeing therefore $\frac{2}{3}$ of B, are to $\frac{2}{3}$ of D, as B to D, (by the preceding) it may be concluded that A : C : : B : D.

PROP. XVII.

If A : B : : C : D, then by diviſion A leſs B : B : : C leſs D : D.

Demonſt. If A contains $\frac{5}{3}$ of B, C will alſo contain $\frac{5}{3}$ of D; then as A leſs B, is equal to $\frac{2}{3}$ of B; ſo C leſs D, is equal to $\frac{2}{3}$ of D; but if A is leſs than B, A leſs B will be a negative quantity, that is, leſs than nothing.

PROP. XVIII.

If A : B : : C : D, then by compoſition A more B : B : : C more D : D.

Demonſt. If A contains $\frac{5}{3}$ of B, C will contain $\frac{5}{3}$ of D; then as A more B, will contain $\frac{8}{3}$ of B; ſo C more D, will contain $\frac{8}{3}$ of D.

PROP. XXII.

If A : B : : C : D, and B : E : : D : F, then b equality of ratio, A : E : : C : F.

Demonſt. If A is the third part of B, C wi be the third part of D; and if B is the fourth par of E, D will be the fourth part of F; therefor as A will be the third part of $\frac{1}{4}$ of E, ſo C will b the $\frac{1}{3}$ of $\frac{1}{4}$ of F.

PROP. XXIII.

If A : B : : E : F, and B : D : : C : E, then b interrupted ratio, A : D : : C : F.

Demonſt. If A is $\frac{1}{3}$ of B, E will be $\frac{1}{3}$ of F and if B is $\frac{1}{4}$ of D, C will be $\frac{1}{4}$ of E; then as A will be $\frac{1}{3}$ of $\frac{1}{4}$, or $\frac{1}{12}$ of D; ſo C will be $\frac{1}{4}$ of $\frac{1}{3}$ or $\frac{1}{12}$ of F.

PROP. the Laſt.

If A : B : : C: D, then by inverted ratio, B A: : D : C.

Demonſtrat. If A is $\frac{1}{4}$ of B, C will be $\frac{1}{4}$ D; therefore as B will be four times A, ſo D will be four times C.

N. B. The preceeding Demonſtrations are univerſal, tho' they ſeem appropriated to particular caſes; and may in the ſame manner be applied in all caſes, both for commenſurable and incommenſurable quantities; for what is here ſaid of third, or fourth parts, may be as well obſerved of any other aliquot parts.

BOOK.

BOOK VI.

DEFINITIONS.

1. A Rectilinear Figure is ſaid to be inſcribed in a curvilinear, when all the angles of the one fall exactly into the boundaries of the other; and the rectiliniar figure circumſcribes a curvilinear, when all the lines of the firſt, if it be a ſurface, or all its planes, if a ſolid, touch thoſe of the laſt.

2. If all the angles of the triangle A B C are equal to the angles of the triangle E D F, that is, if the angle A be equal *Fig.* 1. to the angle D; the angle B to E; and C to F; the two triangles are call'd ſimilar. As for other figures, they are ſaid to be ſimilar, when the triangles of which the one is compoſed, are ſimilar to the triangles which make up the other. Thus the rectilinear figure A is ſimilar to the rectilinear figure B, if the three *Fig.* 2. triangles which compoſe the figure A, are ſimilar to the three triangles of the figure B. In the ſame manner the ſegment C is ſimilar to the ſegment D, if the triangle C is ſimilar to the triangle C.

3. Homologous ſides, are thoſe which in ſimil triangles are oppoſite to equal angle *Fig.* 1. Thus the angle A being equal to th angle D, the ſide C D will be homologous to the ſide E F.

4. A Ratio is ſaid to be compounded of tw others, when its Index or Exponent is the product of thoſe two Exponents multiplied togethe Thus a ſextuple ratio is compounded of doubl and triple ratio; for 2, the exponent of doub ratio, multiplied by 3, the exponent of tripl produces 6, the exponent of ſextuple ratio.

5. If the exponent of any ratio be multiplie into itſelf, it will produce a ratio double to th firſt. Thus quadruple ratio is the double of double ratio; becauſe multiplying 2. the exponent double ratio, by itſelf, it produces 4, the exponen of quadruple ratio. So alſo the ratio whoſe exponent is $\frac{1}{4}$, is double to that whoſe exponent is $\frac{1}{2}$.

6. If the exponent of any ratio be multiplie by the exponent of its double ratio, the produ will be its triple ratio. Thus the ratio, who exponent is 8, will be triple to the ratio who exponent in 2. In like manner the ratio who exponent is $\frac{1}{27}$, will be triple to the ratio who exponent is $\frac{1}{3}$.

LEMMA.

In a ſucceſſion of quantities, as A, B, C, the ratio of the firſt to the laſt will be compoſed of th intermediate ratio's; that is, the ratio of A to C will be compounded of the ratio of A to B, and of B to C.

Demonſt. If A is the third part of B, and B the fourth part of C; A will be the $\frac{1}{3}$ of $\frac{1}{4}$, that is, $\frac{1}{12}$ of C.

COROLLARY.

From whence it follows, that if ſeveral magnitudes as A, B, C, D, E, have their ſame ratio; that is to ſay, if A : B : : B : C, and B : C : : C : D, *&c.* the ratio of A to C, will be double the ratio of A to B; and the ratio of A to D will be triple the ratio of A to B; becauſe the ratio of A to C, is compounded of the ratio's of A to B, and of B to C; and the ratio of A to D is compounded of the ratio's of A to C, and of C to D, or of A to B.

PROP. I.

If the triangles A B C, B C D, are of the ſame height, that is to ſay, ſtand between the ſame parallels, the triangle A B C will be to the triangle B C D, as the baſe A B to the baſe B D. *Fig.* 3.

Demonſt. Suppoſe the baſe A B, to be but a third part of the baſe B D; then drawing the lines C E, C F, to the points E F, which ſhall divide the line B D into three equal parts, and it will appear, that the triangle A B C will be equal to the triangle B C E, (by 37, 1.) which is but the third part of the triangle B C D.

In the ſame manner, it is demonſtrable, that if the baſe A B were but the fourth part of the baſe B D, the triangle A B C would be but a fourth part of the triangle B C D, *&c.* The ſame proof extends likewiſe to Parallelograms.

Prop. II.

If the line D E be drawn parallel to the baſe A C, of the triangle A B C, it will cut
Fig. 4. the ſides A B, B C, proportionally; that is to ſay, the part A D will be to the part B D, as the part C E to the part E B. Draw A E and D C.

Demonſt. The triangles D E A, D E C, having the ſame baſe D E, and being between the ſame parallels C E, A C, will be equal, (by 27. 1.) therefore the triangle A D E will be to the triangle D E B, as the triangle D E C to the ſame triangle D E B, (by 7. 5.) but the triangle A D E is to the triangle D B E, as A D to D B, (by 1. 6.) as is likewiſe the triangle C D E to the triangle D B E, as C E to E B; therefore A D : D B :: C E : E B.

Prop. IV.

If the triangle A B C is ſimilar to the triangle D E F, the ſide D E will be to the ſide
Fig. 5. A B, which is homologous to it, as the ſide F E to its homologous ſide C B. Upon E D take the line E G, equal to A B; and upon E F take E H, equal to B C; and draw G H.

Demonſt. The triangle E G H, will be equal in every reſpect to the triangle A B C, (by 4. 1.) and the angle G will be equal to the angle A, or to the angle D; therefore (by *Cor.* to 15. 1.) the line G H will be parallel to the line D F, and

conſe

conſequently (by the preceding) D G : G E : : F H : H E; then by compoſition, (by 18. 5.) D E : G E, or A B : : F E : H E, or B C.

PROP. V.

If the ſides of the triangle A B C are proportional to the ſides of the triangle D E F; that is to ſay, if as A B : D E : : B C : *Fig.* 6. E F and : : A C : D F, the two triangles are equiangular. Make upon D F the triangle D G F, ſo that the angle D may be equal to the angle A, and the angle F to the angle C.

Demonſt. Since the triangle D F G is ſimilar to the triangle A B C, D F : D G : : A C : A B, (by 7. 5.) or : : D F : D E, therefore D G is equal to D E; and in the ſame manner F E will be equal to F G; and the triangles D E F, D F G, will be equal in every reſpect, (by 8. 1.) therefore the triangle D F E, will be equiangular to the triangle A B C, as is the triangle D F G.

PROP. VIII.

If the triangle A B C has one angle right, as B, the perpendicular B D ſhall divide it into two triangles, both ſimilar to *Fig.* 7. it; becauſe they will each have one right angle as the given triangle; and will likewiſe each have one angle common with it.

PROP. IX.

To divide the line A B into any number of equal parts. Draw the line *Fig.* 8. B C, at pleaſure, and with any opening of the compaſs ſet off ſo many equal parts upon the line B C, as you intend to divide the line A B into; for example, 4: then from the point G, which determines the laſt, draw the line G A, to which drawing parallels thro' the ſeveral diviſions of the line C B, to wit, F H, E L, D M, and the points H, L, M, ſhall divide the line A B into 4 equal parts.

Demonſt. As the line B D : to the line B G : : B M : B A, (by 2. 6.) therefore as B D is the $\frac{1}{4}$ of B G, ſo B M will be the $\frac{1}{4}$ of B A, *&c.*

To make the work the eaſier, draw A N, parallel to B C, and ſet off upon A N the ſame diviſions as upon B C; then drawing your lines from the marks on one, to the correſponding marks on the other, ſuch lines will be parallel (by 31. 1.) and the given line will be divided as required.

PROP. X.

Fig. 9. To divide the line A B in proportion as the line E is already divided. Draw the parallels A D, and B C, equal to the line E, and lay down the diviſions of the line E, upon the line A D, beginning at the point A; do the ſame on the line C B, beginning at the point C; then drawing thro' the marks H L, the lines H L, they ſhall divide the line A B, as the lines A D, and B C, (by the preceding) or as the given line E was divided.

PROP. XI.

Fig. 10. To find a third proportional to the lines A and B. From the ſame point E draw two lines at pleaſure, making any angle; then taking E O, equal to the line A, and the lines O H, and E K, equal to the line B, draw H L parallel to O K; ſo ſhall the line K L be the third proportional ſought.

Demonſt. E O : O H : : E K : K L, (by 2. 6.) therefore A : B : : B : K L.

Prop. XII.

The ſame muſt be done, if a fourth proportional to the lines A B C be demanded. For making E O equal to A, O H equal to B, and E K equal to C, K L will be the fourth proportional.

Fig. 10.

Prop. XIII.

To find a mean proportional between the line A and the line B. Upon the ſame line lay down A B equal to A, and B C equal to B; then having deſcribed a ſemicircle upon A C, draw the perpendicular B D, which will be a mean proportional between A B and B C.

Fig. 11.

Demonſtrat. The angle A D C, being a right angle (by 31. 3.) the perpendicular B D will divide the triangle A D C, into two ſimilar triangles A B D, D B C, (by 8. 6.) therefore A B : B D : : B D : B C.

Prop. XIV.

If the equiangular parallelograms A B are equal, their ſides ſhall be in reciprocal ratio to each other; that is

Fig. 12.

to ſay, as the ſide E F is to the ſide F G, ſo is the ſide H F to the ſide F I.

Demonſt. Since the parallelograms A B are equal, they will have the ſame ratio to a third C, (by 7. 5.) but A : C : : E F : F G, (by 1. 6.) and B : C : : H F : F I; therefore E F : F G : : H F : F I.

In the ſame manner if the ſides of the ſaid parallelograms are in reciprocal ratio, we may conclude them equal; becauſe A will be to C, as B to C.

Prop. XVI.

If the four lines A B C D are proportional, the parallelogram E, upon the firſt and fourth, the two extreams, will be equal to the parallelogram F, upon the ſecond and third, the two means, (by the preceding) becauſe their ſides will be in reciprocal ratio, that is, the ſide A : ſide B :: C : D. *Fig.* 13.

It would be the ſame thing if the lines B C were equal; therefore we ſay, that the ſquare of a mean proportional is equal to the rectangle of the two extreams.

COROL. I.

If the lines A B, C D, cut each other within a circle, at the point E, the rectangle contained under the parts A E, E B, of the one will be equal to the rectangle of the parts C E, E D, of the other. *Fig.* 14.

Demonſt. The triangles A E C, D E B, have the oppoſite angles E equal, (by 15. 1.) and the angles C B reſting upon equal arches, are alſo equal, (by 21. 3.) therefore they are ſimilar, and conſequently (by 4. 6.) A E : E D : : C E : E B; therefore the rectangle contained under A E, E B, will be equal to the rectangle contain'd under D E, C E, (by the preceding.)

COROL. II.

If from the point A without a circle you draw the line A C cutting the circle, and the line A D a tangent to it; the rectangle contain'd under the line A C, and the part A F, without the circle, will be equal to the ſquare of the line A D.

Fig. 15.

Demonſt. The triangles C A D, A D F, are ſimilar, having the angle A common, and the angle C equal to the angle A D F, (by 32. 3.) therefore A C : A D : : A D : A F, (by 4. 6.) and conſequently the ſquare of the mean proportional A D will be equal to the rectangle of the two extreams A C, A F, (by the preceding.)

PROP. XIX.

If the triangles A and B are ſimilar, the triangle A will be to the triangle B in a duplicate ratio to the ratio between their homologous ſides. That is to ſay, if the baſe L M is double the baſe G H, the triangle A will be quadruple the triangle B.

Fig. 16.

Demonſt. Dividing equally the lines L M in the point D, and L C in the point E, and drawing D E, D C, it will appear, that the triangle E L D is equal to the triangle B, (by 4. 1.) becauſe the angle L is equal to the angle G, and the ſides L D L E, equal to the ſides G H and G F. But the triangle L E D, is $\frac{1}{4}$ of the triangle L C M, being but $\frac{1}{2}$ of the triangle L C D, (by 1. 6.) which is itſelf but $\frac{1}{2}$ of the triangle L C M; therefore the triangle *B* is $\frac{1}{4}$ of the triangle L C M.

In the ſame manner it may be demonſtrated, that if L M is triple G H, or quadruple, the triangle A would accordingly be nine, or ſixteen times bigger than B, *&c.*

PROP. XX.

Fig. 17. The ſimilar polygons A and B are alſo in a duplicate ratio to their homologous ſides, becauſe they are compoſed of ſimilar triangles.

PROP. XXIII.

Fig. 18. If the parallelograms A and B are equiangular, the ratio of the parallelogram A to the parallelogram B will be compoſed of the ratio of the line C D, to the line D E, and of the ratio of the line G D, to the line D F.

Demonſt. If C D is $\frac{2}{3}$ of D E, the parallelogram A will be $\frac{2}{3}$ of the parallelogram H, (by 1. 6.) and if G D is $\frac{1}{2}$ D F, the parallelogram H will be $\frac{1}{2}$ the parallelogram B, and the parallelogram A will be $\frac{2}{3}$ of $\frac{1}{2}$, or $\frac{2}{6}$, *i. e.* $\frac{1}{3}$ of the parallelogram B.

PROP.

Prop. XXX.

If the line A B be divided in the point C, ſo that the rectangle contained under A B, and B C, be equal to the ſquare of A C, (by 11. 2.) then is ſuch line divided in extream and mean proportion; becauſe A B : A C : : A C : C B, (by 16. 6.) *Fig.* 19.

BOOK XI.

DEFINITIONS.

1. A *Body* or a *Solid* is a magnitude wherein we conſider length, breadth, and thickneſs.

2. A line is ſaid to be right upon a plane, when it makes right angles with all the lines of the plane. Thus the line A B will be right to the plane C D E F, if the angles C B A, D B A, E B A, *&c.* are right. From this Definition it follows, that two lines cannot be right to the ſame plane, and fall upon the ſame point. *Fig.* 1.

3. If the planes G and D cut each other, ſome of their points, as A B, will be found in both planes, and a right line drawn thro' thoſe points will alſo paſs thro' both planes, and is therefore call'd their *common Section.* *Fig.* 2.

4. The angle made by the plane A, with the plane B, is the ſame as would be made by the lines of both, which are perpendicular to their common ſection, as D G C, C E F. *Fig.* 3.

5. Solids are equal in every reſpect, when they are bounded with an equal number of equal ſurfaces, and diſpoſed in the ſame manner.

6. Similar Solids are bounded with an equal number of ſimilar ſurfaces, and diſpoſed in the ſame manner.

7. A ſolid angle is form'd by the concourſe or meeting of more than two planes in the ſame point, not making one plane, as B. *Fig.* 4.

8. Parallel planes are ſuch as being infinitely prolong'd will not meet.

9. A *Priſm* is a body bounded with two right-lin'd parallel planes, equal in every reſpect, and by an infinite number of right lines paſſing from the one to the other, as the body A. *Fig.* 5.

10. *Parallelipepidons* are bodies bounded by ſix planes, of which every oppoſite two are parallels, as B. When theſe ſix planes are true ſquares, the parallelipepidon is a *Cube*, as C. *Fig.* 6.

11. A *Pyramid* is a body bounded by one right-lin'd plane, and an infinite number of right lines, drawn from the circumference of the ſaid right lin'd plane, to a point without it, as D. *Fig.* 7.

12. A *Cylinder* is a body bounded by two circular planes, equal, and parallel, and an infinite number of right lines paſſing from the one to the other, as E. The line connecting the centers of the two circles is call'd the *Axis*, as E F. *Fig*. 8.

13. A *Cone* is a body bounded by one circular plane, and an infinite number of right lines, drawn from the circumference, to a point without its plane, as H. The line joining the center to the ſaid point, at the top, or vertex of the Cone, is call'd its *Axis*, as E H. *Fig*. 9.

14. A *Globe*, or *Sphere*, is a body whoſe extream parts are all equally diſtant from a certain point within it, which point is call'd its center, as P. *Fig*. 10.

PROP. VI.

Fig. 11. If the lines A B, C D, are right upon the ſame plane, they are parallel.

Demonſt. They will be in the ſame plane, to wit, that which would be deſcribed by the line C A, if remaining right upon the plane, they both meet the line C A: farther, they will be perpendicular to the line A C, (by *Def*. 1.) therefore they will be parallel.

Prop. VIII.

If the lines A B, C D, are parallel, and A B right upon a plane, C D ſhall be alſo right upon the ſame plane; otherwiſe, *Fig.* 11. one might imagine the line C E right upon the plane, which would then be alſo parallel to the line A B, (by the preceding) which cannot be.

Prop. IX.

If the lines A and B are parallel to a third C, they will be parallel to one another; for imagining a plane, upon which the *Fig.* 12. line C is right, the lines A and B, which are parallel to the line C, will be right upon the ſame plane, and conſequently parallel to one another.

Prop. X.

If the line A B is parallel to the line C D, and the line B E parallel to the *Fig.* 13. line D F, the angle A B E will be equal to the angle C D F.

Demonſt. The lines A B, C D, and E B, D F, being equal, the lines A C, E F, will be equal, and parallel to a third B D, and conſequently to one another; therefore the lines A E, C F, will be alſo equal, (by 33. 1.) and all the ſides of the triangle A B E, will be equal to thoſe of the triangle C D F, and conſequently (by 8. 1.) the angle A B E equal to the angle C D F.

PROP. XVI.

If the parallel planes A and B are cut by a third plane C D F E, the common *Fig.* 14. ſections C D, E F, will be alſo parallel; becauſe they will be in the ſame plane, and can never meet, any more than the two planes A and B.

PROP. XVIII.

If the line A B is right upon the plane O A F D, all the planes which can paſs thro' the *Fig.* 15. line A B will be perpendicular to the ſame plane; becauſe the line A B, which determines the angle every ſuch plane will make with the plane O A F D, ſtands itſelf at right angles with the ſaid plane, (by *Def.* 2.)

PROP. XXIV.

If a body is bounded with ſix planes, of which every oppoſite two are parallel, as A *Fig.* 16. and B, C and D, E and F, thoſe oppoſite planes are equal, and ſimilar parallelograms one to another.

Demonſt. Since the parallel planes A and B are cut by the plane D, the common ſections G H, and K I, will be parallel, (by 16. 11.) ſo will alſo I H, K G, and the figure H I K G will be a parallelogram. In the ſame manner the figures G K L N, K L M I, I M O H, O H G N, O N L M, will be parallelograms; and conſequently the lines anſwering each other are equal, as G K, N L; O M, H I; and G H, O N; M L, K I. Farther, the angles which anſwer each other are equal, as the angle H G K to the angle O N L, becauſe the lines H G, O N, G K, N L, are parallel: Therefore the planes D and C, and the other correſpondent ones, are equal, and ſimilar parallelograms.

Prop. XXVIII.

If the parallelipepidon A be cut by the plane B C, paſſing thro' the diagonals of two oppoſite planes, the two parts will become two equal priſms, ſince they will be bounded with an equal number of equal and ſimilar planes, diſpoſed in the ſame manner.

Fig. 17.

Prop. XXIX.

If the parallelipepidons A B E F D H G and A B N M D O L have the ſame baſe A B D, and the ſame heighth D H, they are equal; becauſe they have the priſm A B D H G N M common, and the priſm A B N M E F is equal to the priſm D G H O L, therefore (by *Ax.* 3.) the two parallelipepides are equal.

Fig. 18.

Prop. XXXIII.

If the parallelipedidon A is ſimilar to the parallelipepidon E, and the ſide B C be double the ſide F G, which is homologous to it, the parallelipedon A will be eight times the bigneſs of the parallelipepidon E.

Fig. 19.

Demonſt. Since B C is double F G, B D will be alſo double F H, and D L double H I: If then you divide the parallelipepidon A, in its length, breadth, and depth, into two equal parts, you will make eight parallelipepides, any one of which, as M, having all its planes equal, and ſimilar to the planes of the parallelipepidon E, will conſequently be equal to it, in every reſpect.

In the ſame manner if the line B C was triple the line F G, it might be proved, that the parallelipepidon A would contain twenty-ſeven times the parallelipepidon E; from whence it follows, that ſimilar parallelipepides are in triple ratio, to their homologous ſides; as are alſo ſimilar priſms, which are the halves of ſimilar parallelipepides.

Prop. XXXVIII.

If the plane A is right upon the plane B, and from the point A be drawn a line A B, perpendicular to their common ſection C D, it will be right upon the plane B. *Fig*. 20.

Demonſt. If from every point of the line C D were drawn lines all right to the plane B, they would conſtitute the plane A, and one of thoſe lines muſt be the line A B.

BOOK XII.

PROP. I.

IF the Polygons R and O, inſcribed in circles, are ſimilar, the ſide A E will be to the ſide C F as the diameter A B to the diameter C D; becauſe the triangles A B E, C D F, are ſimilar. *Fig.* 1.

The ſame may be demonſtrated of all the lines which bound a ſimilar arch.

PROP. II.

The more ſides belong to a polygon, either inſcrib'd in a circle or circumſcrib'd about it, the leſs it differs from a circle; therefore a polygon having an infinite number of ſides, hardly differs at all from a circle: from whence it follows, that as polygons are in a duplicate ratio of the radii of the circles in which they are inſcribed, (by the preceding) ſo circles alſo are in a duplicate ratio to their own radii. *Fig.* 2.

COROLLARY.

Cylinders may be alſo taken for Priſms; from whence it may be concluded, that Cylinders are in a triple ratio of their homologous ſides, as the diameters of their baſes, when they are ſimilar.

Prop. III.

If the pyramid A, and the pyramid B, have the ſame baſe DCE, and the ſame heighth; *Fig.* 3. that is to ſay, if the line AB, which joins their vertexes, be parallel to D E, one of the lines of their baſe, the two pyramids are equal.

Demonſt. The triangle D C E is compoſed of an infinite number of lines parallel to the line D E. If from the ends of theſe lines other lines be drawn to the point A, they will conſtitute an infinite number of triangles, compoſing the pyramid A: In like manner the pyramid B would be compoſed, if the ſaid lines be brought to the point B. But every one of thoſe triangles which compoſe the pyramid A will be equal to thoſe which form the pyramid B, (by 37. 1.) therefore the whole pyramid A will be equal to the whole pyramid B.

Prop. VII.

The triangular pyramid D E F C is but th third part of the priſm A B C D E F, *Fig.* 4. which has the ſame baſe, and the ſam heighth; becauſe the whole priſm contains two more pyramids, *viz.* A B F C, an A D F C, which having equal baſes A B F, an A D F, and the ſame vertex C, are equal, (by th preceding) and one of them, *viz.* A B F C, is equa to the pyramid D E F C, (by the preceding) becauſe it has both the ſame baſe, and the ſam heighth as the priſm.

It is the ſame thing with pyramids which are not triangular, becauſe they may be divided into triangular pyramids. From whence it follows, that ſimilar pyramids are in a triple ratio of their homologous ſides; and conſequently cones (which may be taken for pyramids of an infinite number of ſides) as they alſo are a third part of cylinders, of the ſame baſe, and heighth, are likewiſe in a triple ratio of their homologous ſides.

PROP. the Laſt.

A Globe may be conſidered as compoſed of an infinite number of pyramids, whoſe vertexes meet at the center of the Globe, their baſes compoſing its ſurface, and their ſides forming its radii. So that each pyramid compoſing the globe A, being to each pyramid compoſing the globe B, in a triple ratio of the radius A C, to the radius B D; the globe A will be to the globe B in a triple ratio of the radius A C, to the Radius B D.

Fig. 5.

The ſame may be ſaid of all other ſimilar bodies.

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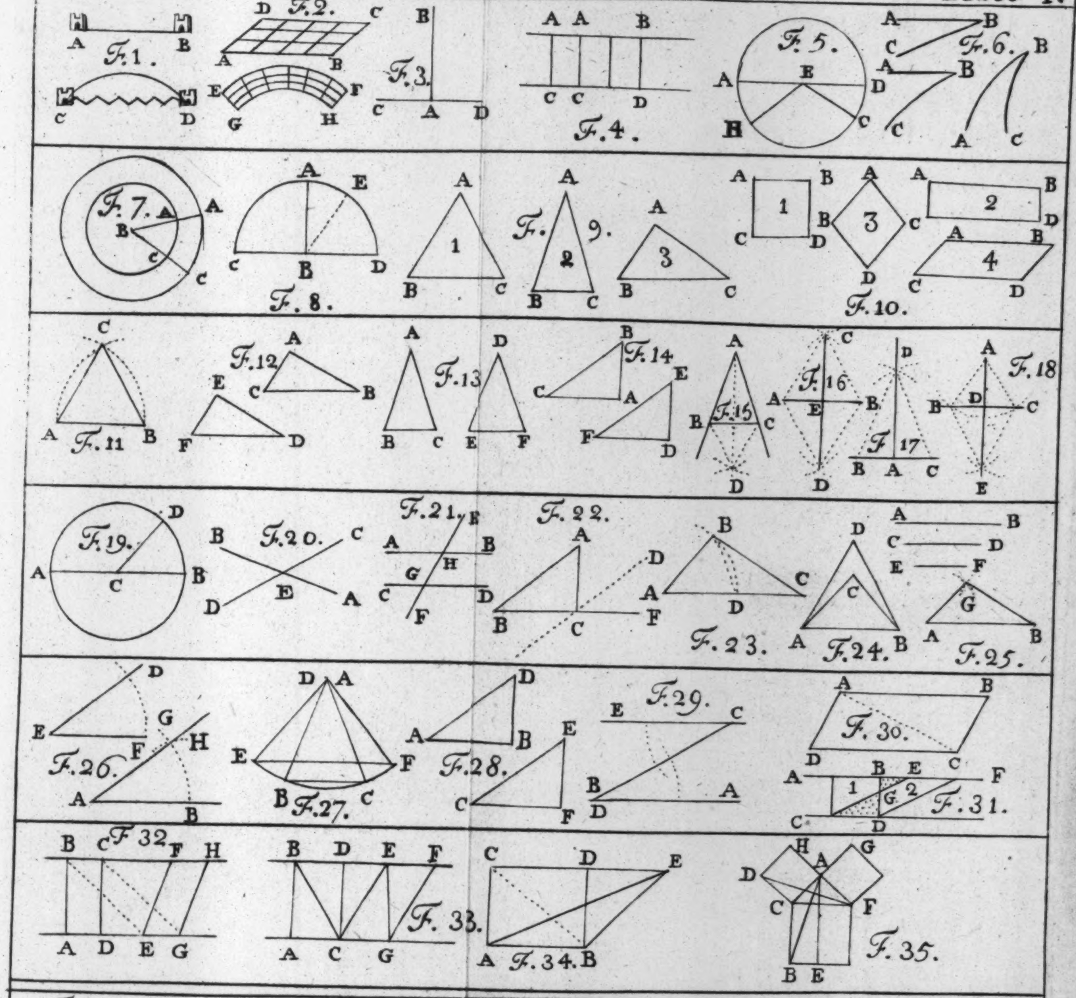

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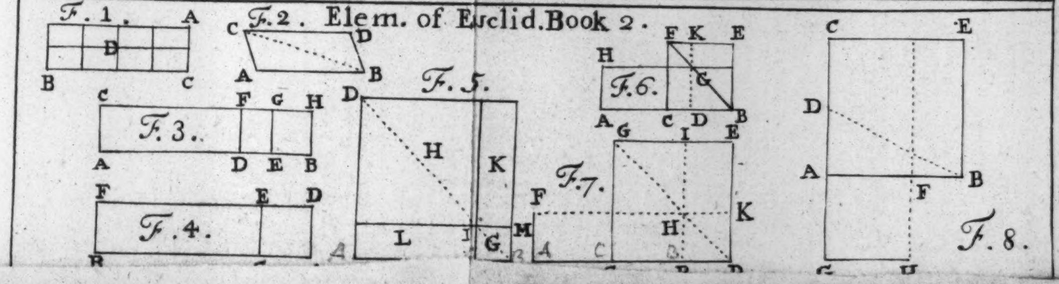

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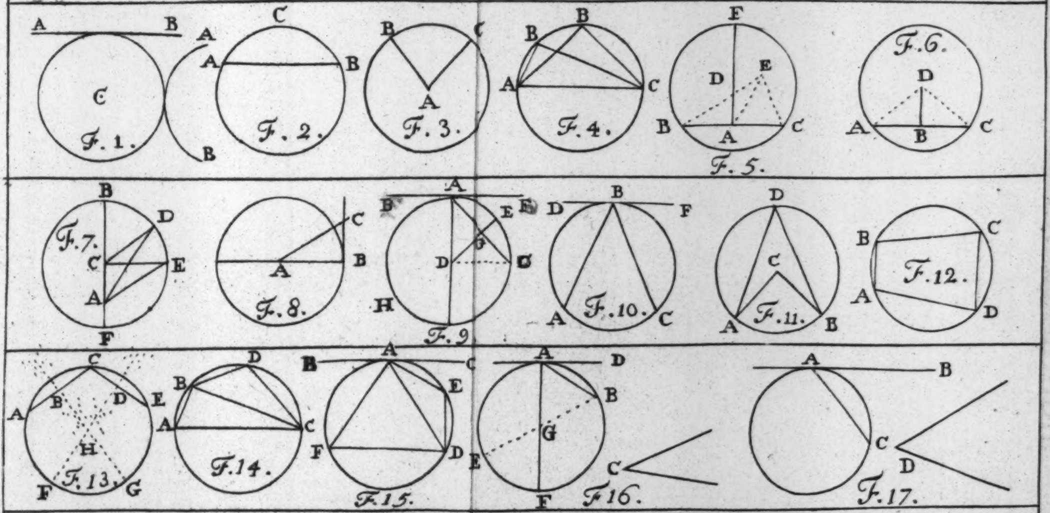

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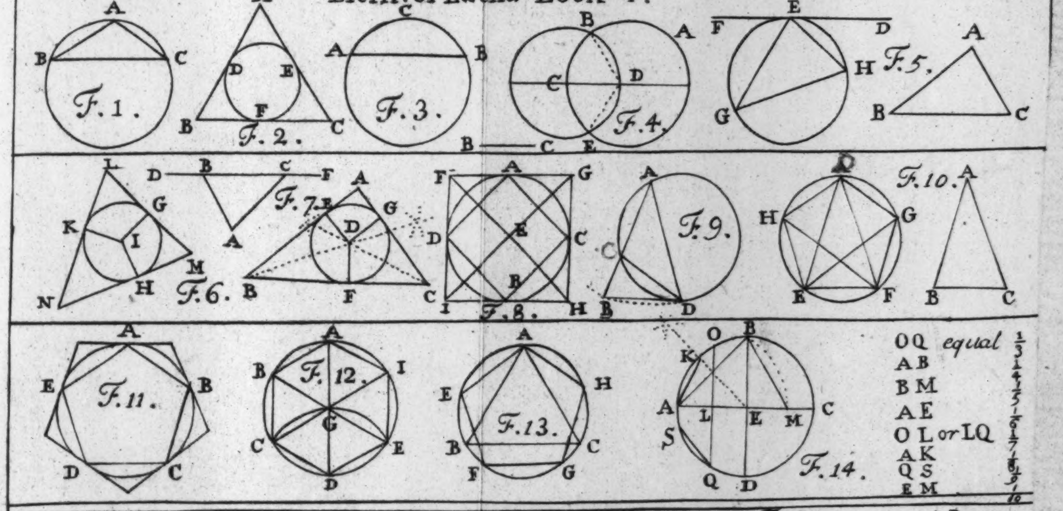

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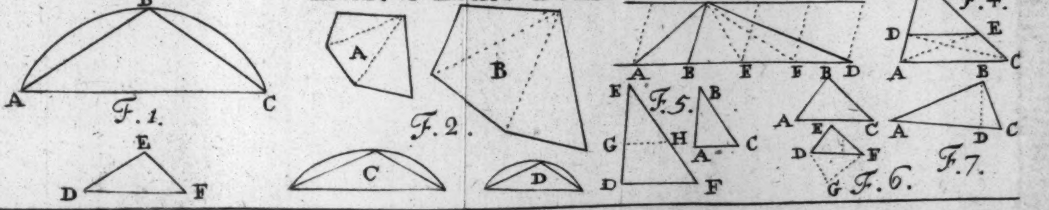

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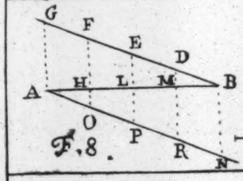

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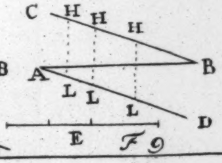

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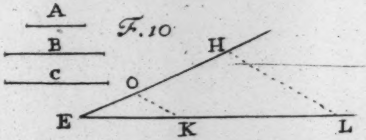

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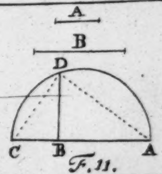

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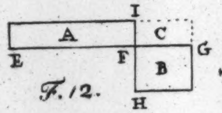

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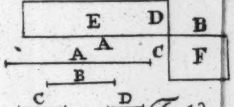

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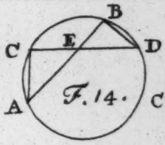

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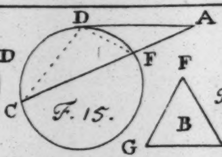

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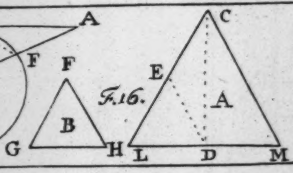

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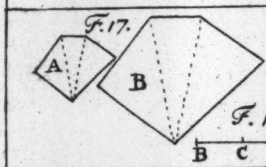

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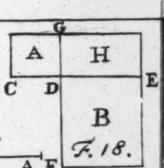

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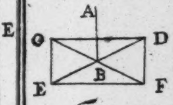

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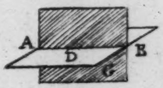

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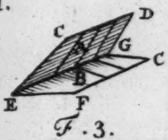

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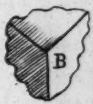

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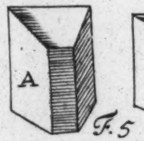

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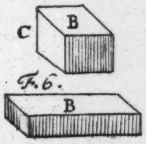

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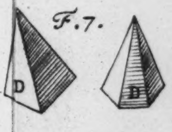

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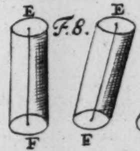

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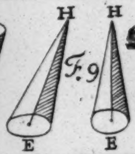

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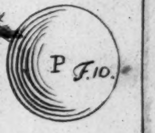

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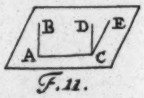

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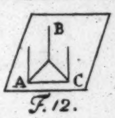

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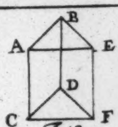

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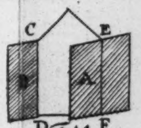

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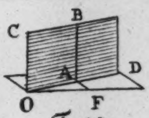

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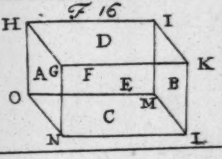

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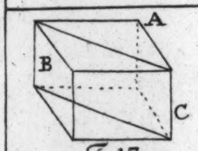

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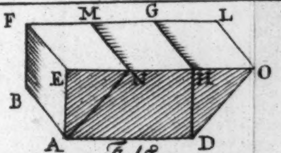

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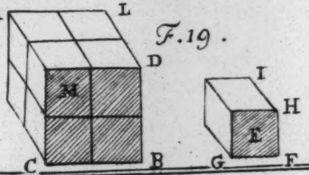

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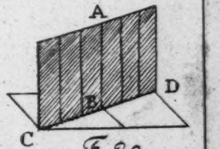
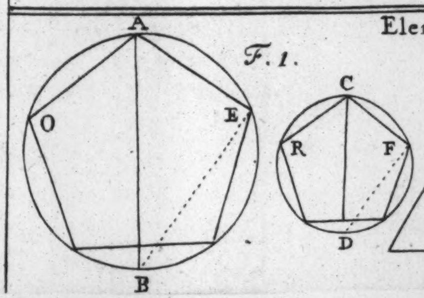
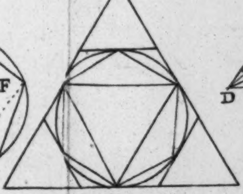


F. 20

Elem. of Euclid Book 12.



F. 1.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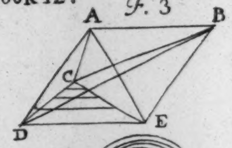
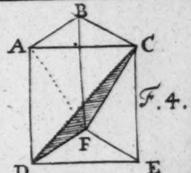


F. 3



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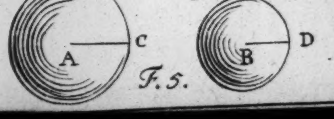

F. 5.

TRIGONOMETRY.

TRIGNOMETRY is a Science by which we learn the reſolution of Triangles; that is, to find the angles and ſides unknown, by the angles and ſides known.

PART I.

The Conſtruction of the Tables of Sines, Tangents, and Secants.

DEFINITIONS.

1. AN Arch is a part of the circumference of a circle, as A B C; *Fig.* 1. the right line A B extending from the one end of the arch to the other, is call'd the Cord, or ſubtendent of the arch.

The whole circumference of every circle is ſuppoſed to be divided into 360 degrees, each degree into 60 minutes, and each minute into 60 ſeconds, *&c.*

2. What

Fig. 2.

2. What an arch wants of 90 degrees, is call'd its *complement*; and its defect of 180 degrees, is call'd its *ſupplement*. Thus the arch A C is the complement of the arch A B, if B A C contains 90 degrees; and A D is its ſupplement, if the whole figure is a ſemicircle.

Fig. 3.

3. The right ſine of an arch is a right line falling from the one end of the arch perpendicularly on the diameter that terminates in the other end. Thus B C is the right ſine of the arch B A, or of its ſupplement B F.

4. The ſine complement, or co-ſign of an arch, is the right ſine of its complement. Thus B E, which is the right ſine of B G, the complement of B A, is call'd the ſine complement, or co-ſine of B A; and ſince B E is always equal to D C, therefore D C is call'd the ſine complement of the arch B A. *V. Fig.* 3.

5. The verſed ſine of an arch is that part of the diameter contain'd between the arch and its right ſine. Thus C A is the verſed ſine of the arch B A, and C F of the arch B F. *V. Fig.* 3.

6. The Radius of a circle, as G D, is call'd the ſine total, becauſe it is the greateſt right ſine. *V. F.* 3.

Fig. 4.

7. The Tangent of an arch is a right line touching one end of the arch, and drawn without the arch, parallel to the right ſine till it meets the radius produced thro'

the other end: The radius ſo produced is call'd a Secant. Thus A B is the Tangent of the Arch A C, and D B the Secant.

Thus the Tables of Sines, Tangents, and Secants, being valued by the proportion they bear to the radius of every circle, and that proportion being in numbers often irrational, the Tables cannot be perfectly exact; but that there may be no ſenſible error in their application to practice, the radius of each circle is divided into 10000 equal parts; and the Tables ſhew how many of ſuch parts are contain'd in every Sine, Tangent, and Secant, neglecting the fractions.

LEMMA I.

In every four-ſided figure A B D E, inſcribed in a circle, the rectangle made by the diagonals A D, E B, is equal to the two rectangles made by the oppoſite ſides of the ſaid figure, to wit, the rectangle made by E A, D B, and the rectangle made by E D, A B. Draw B C, ſo that the angle A B C may be equal to the angle D B E. *Fig.* 5.

Demonſtrat. The triangle A B C is ſimilar to the triangle E D B; for beſides the equal angles A B C, D B E, the angle B E D is equal to the angle B A D, (by 21. 3.) Therefore (by 4. 6.) A B : A C : : E B : E D, and conſequently the rectangle of the extreams A B, E D, will be equal to the rectangle of the means A C, E B, (by 16. 6.) In the ſame manner the rectangle upon A E, B D, will be equal to the rectangle upon D C, E B; becauſe the triangles B C D, B A E, are ſimilar, and conſequently the rectangle upon E B, and the whole line A D, will be equal to the two rectangles upon A B, E D, and upon A E, B D.

LEMMA II.

If the content of the rectangle A B C D, and one of its ſides A B, be known, the *Fig.* 6. other ſide A D, may be known, by dividing the value of the rectangle, by the ſide known.

LEMMA III.

If as A : B : : C : D, and A, B, C, be known, D may be found, by multiplying B by C, and dividing the product by A, ſince that product is equal to the product of A by D, (by 16. 6.)

PROP. I.

Fig. 7. If it is known how many parts of the diameter A C are contained in the cord of an arch A B, the content of its ſupplement B C may be alſo found.

Demonſt. The angle B being right, the ſquare of the diameter A C will be equal to the ſquare of the line A B, and the ſquare of the line B C, (by 47. 1.) and conſequently if the ſquare of the line A B be ſubtracted from the ſquare of the diameter A C, the ſquare of the line B C will remain; the root of which being extracted, will give the line B C.

PROP. II.

The cord B C of an arch being given, the cord C A of an arch double C B may be found. Draw the diameter B D, and the lines D A, D C. *Fig.* 8.

Demonſt. D A, D C, may be found by the foregoing. Multiply A B by C D, and C B by A D, the ſum of the two products will be equal to the rectangle made upon B D and A C, (by *Lemma* 1.) therefore dividing by B D, you have C A, (by *Lemma* 2.)

PROP. III.

The cord A B being known, the cord D B of an arch triple to A B may be found. Draw A C, and C D, equal to A B; draw alſo A D, C B, which may be known as before. *Fig.* 9.

Demonſt. Multiply the cord A D by the cord B C, and the product will be equal to the two rectangles upon D C, A B, and upon C A, D B, (by *Lemma* 1.) then ſubtracting the rectangle upon C D, A B, the remainder will be the rectangle upon C A, D B, which being divided by C A, gives D B.

In the ſame manner may be found the cord of an arch quadruple, quintuple, ſextuple, &c.

PROP. IV.

The cord of an arch of 60 degrees A B is equal to the radius C A. *Fig.* 10.

Demonſt. The angle C contains 60 degrees, the two others A, B, contain what remains to make up the two right angles, that is, 120 degrees, (by 16. 1.) and becauſe they are equal, each muſt contain 60 degrees, (by 5. 1.) as the angle C, and conſequently the line A B will be equal to the lines C A, or C B, (by 6. 1.)

PROP. V.

Fig. 11. Half the cord A B, is the right ſin of half its arch A C B.

Demonſt. If the line A B be divided equall in the point E, the triangles A E D, B E D, wil be equal in very reſpect, (by 8. 1.) and conſequently A E will be the right ſine of A C, (b *Def.* 3.) which is the half of A C B, ſince th angles A D C, B D C, are equal.

COROLLARY.

The cord of 60 degrees being equal to the radius, contains 100000 parts, therefore the righ ſine of 30 degrees, being half the cord of 60 degrees, will contain 50000.

PROP. VI.

Fig. 12. If the right ſine A B, of an arch o 30 degrees A D, be known, the co A D may be alſo known.

Demonſt. Subtract the ſquare of the ſine A from the ſquare of the radius A C, the remaind will be the ſquare of the line B C, (by 47. 1 which being ſubtracted from the radius C D, wi give the verſed ſine B D; add therefore the ſqua of B D, to the ſquare of B A, and the ſum will be the ſquare of the cord A D (by 47. 1.)

COROL. I.

Having the cord A D, the half thereof will be the right ſine of 15 degrees, the cord of which may be found by the foregoing, and its half will again be the right ſine of 7 degrees 30 minutes. Thus deſcending by continual halving, you will find that the right ſine of $52''$. $44'''$. $3''''$. $45'''''$. is 2554. You muſt obſerve, at the ſame time, that the ſine of $1'$. $45''$. $28'''$. $7''''$. $30'''''$. is double the ſine of $52''$. $44'''$. $3''''$. $45'''''$. ſo that the ſine is to the ſine as the arch to the arch; therefore without danger of miſtake, you may ſay, as $52''$. $44'''$. $3''''$. $45'''''$. are to $60''$. ſo 2554 to the ſine of $60'$. which will conſequently be 2909.

COROL. II.

Having thus found the right ſine of 1 minute, you muſt double it to get the cord of 2 minutes, which by *Prop.* 2. will give the cord of an arch double, or the cord of $4'$. of which the half will be the ſine of $2'$. and by *Prop.* 3. it will give you the cord of a triple arch, or of $6'$. the half of which will be the ſine of $3'$. Laſtly, this ſame cord of $2'$. will give the cord of an arch quadruple, quintuple, ſextuple, &c. or of $8'$. of $10'$. of $12'$. &c. the half of which will be the ſine of $4'$. of $5'$. of $6'$. &c.

PROP. VII.

Having found the right ſines of all the arches, you will have their tangents, and their ſecants. For Example, you may have the tangent C E, if you know the ſine D B, and the ſine complement A B. *Fig.* 13.

Demonſt. The triangles A B D, A C E, are rectangled, and have the angle A common; therefore they are ſimilar, and conſequently the ſine complement A B, will be to the right ſine B D, as the radius A C to the tangent C E, (by 4. 6.)

In the ſame manner for the ſecants, the ſine complement A B will be to the radius A C, as the radius A D to the ſecant A E, (by 4. 6.)

Note, *To the end that the fractions neglected in the beginning, may not cauſe any great error in the progreſs, it will be proper to take them into the produce, ſo as to be conſtantly within the half unit, in the loweſt place.*

PART II.

LOGARITHMS.

LOGARITHMS are artificial Numbers, made uſe of inſtead of the natural ones, by means of which the operations of Multiplication, and Diviſion, are performed by Addition and Subtraction; and the extraction of the Root of any power by a ſingle Diviſion of the Logarithm of the power by the index or exponent of ſuch power; thus dividing it by 2, the quotient is the Logarithm of the ſquare root, and by 3, that of the cube root, *&c.*

DEFINITIONS.

1. Arithmetical Progreſſion, is a ſucceſſion of numbers equally exceeding each other, as 0, 1, 2, 3, 4, 5, *&c.* or 1, 3, 5, 7, 9, *&c.*

2. Geometrical Progreſſion, is a ſucceſſion of numbers, of which the firſt is to the ſecond, as the ſecond to the third, and as the third to the fourth, *&c.* as 1, 2, 4, 8, or 1, 3, 9, 27.

3. If exactly under the geometrical progreſſion A, beginning with 1, you place the arithmetical progreſſion B, beginning with 0, the numbers of

the arithmetical progreſſion will be the logarithms of the correſponding numbers in the geometrical progreſſion. For example, the third number of the arithmetical, will be the logarithm of the third number of the geometrical; that is, the number C is the logarithm of the number D, ſo is E of F, and G of H.

D F H
A ∺ 1, 2, 4, 8, 16, 32, 64.
B ÷ 0, 1, 2, 3, 4, 5, 6.
C, E, G.

LEMMA I.

In every geometrical progreſſion A, the numbers immediately following each other, have the ſame common ratio between them; but thoſe which are ſeparated by the interpoſition of one term or place, as B C, have a ratio, double to the general one; and thoſe which are ſeparated by the interpoſition of two places, as B, D, have a triple ratio, *&c.* (by *Lem.* 1. 6.)

A ∺ 1, 2, 4, 8, 16, 32.
B, C, D.

COROL. III.

From hence it follows, that if two numbers in a geometrical progreſſion are at the ſame diſtance from each other, as two other numbers, they will have the ſame ratio; and if they have the ſame ratio, they are at the ſame diſtance.

LEMMA

LEMMA II.

In every geometrical progreſſion A, whoſe firſt term is unity, if the number B be multiplied by the number C, the product ſhall be a number in the ſame progreſſion, as far diſtant from the number C, as the number B is from unity, to wit, D.

A ∺ 1, 2, 4, 8, 16, 32, 64, 128.
B, C, D.

Demonſt. If multiplying B by C produces D, it muſt be 1 : B : : C : D, therefore (by *Lemma* preceding) B will be as far diſtant from unity, as D from C.

On the contrary, if D be divided by B, the quotient will be C, as far diſtant from D, as the diviſor B from unity; becauſe in every diviſion, the dividend is to the quotient, as the diviſor is to unity.

LEMMA III.

In every arithmetical progreſſion commencing with 0, if the number marked C, be added to the number D, the ſum will be E; the ſame diſtance from D, as C from 0.

B ÷ 0, 1, 2, 3, 4, 5, 6, 7.
C, D, E.

Demonſt. C will be more or leſs diſtant from 0, according as it contains more or leſs times the common difference of the progreſſion. In the ſame manner E will be more or leſs diſtant from D, according as it contains more or leſs times the ſame common difference above D. But ſince E is the ſum of C and D, it will contain the ſame number of times the common difference more than D, as C does more than 0; therefore E will always be as far diſtant from D, as C from 0.

In the ſame manner, it may be proved, that if, on the contrary, the number C be ſubtracted from the number E, the remainder will be the number D, as far diſtant from E, as C from 0, ſince E contains as many times the common difference above D, as C does above 0.

PROP. VIII.

If in the arithmetical progreſſion B, anſwering the geometrical progreſſion A, the number C, the logarithm of the number D, be added to the number E, the logarithm of the number F; the ſum G will be the logarithm of the number H, and the number H will be the product of the number D, multiplied by the number F.

		D			F	H	
A ∺	1,	2,	4,	8,	16,	32,	64.
B ÷	0,	1,	2,	3,	4,	5,	6.
		C			E	G	

Demonſt. Since in the arithmetical progreſſion, the number C immediately follows 0, if to it be added the number E, the ſum G muſt immediately follow the number E, (by *Lem.* 3.) ſo in the geometrical progreſſion, ſince D immediately follows 1, if it be multiplied by F, the product H ſhall immediately follow F, (by *Lem.* 2.) Therefore ſince E anſwers to F, ſo ſhall G to H, and will conſequently be its logarithm.

In the ſame manner it may be demonſtrated, that if C, the log. of D, is ſubtracted from G, the log. of H. the remainder E, will be the log. of the number F, and the number F, will be the quotient of H, divided by D.

COROLLARY.

The Uſe of Logarithms.

1. If you are to find a fourth proportional to three numbers given A, B, C, having found in ſome Table of Logarithms, the logarithms of the given numbers, add the log. of B and C, and from their ſum E, ſubtract the log. of the number A, and the remainder D, ſhall be the log. of the fourth number required F.

A 1097 —— 3,0402066
B 1078 —— 3,0326188
C 964 —— 2,9840770

E —— 6,0166958
A —— 3,0402066

D —— 2,9764892
F 947 $\frac{1}{8}$

Demonſtrat. To find a fourth proportional, the rule is, to multiply the ſecond and third, and divide the product by the firſt; but in ſubtracting the log. of the number A, from the ſum of the logs. of the numbers B and C, we find the log. of the product of the numbers B and C, divided by the number A; therefore that log. muſt be the logarithm anſwering to the fourth proportional ſought.

2. To find the ſquare root of any number, as 9801, having from any table found its logarithm,

which is 3,9912704, the half thereof, *viz.* 1,9956352, will be the log. of the root ſought, to wit, 99. For adding the log. of 99 to itſelf, it gives the product of 99 multiplied into itſelf, that is its ſquare.

In the ſame manner taking the third part of the logarithm of any number, you have the log. of its cube root; and taking the $\frac{1}{4}$ you have the log. of its biquadrate root, *&c.*

PROP. IX.

The Conſtruction of Logarithms correſpondent to the natural Numbers.

1. We begin with the geometrical progreſſion A, in which the terms increaſe by the ratio of one to ten: for the logarithms of which we ſet down the arithmetical progreſſion B, in which the terms exceed each other by 1000000. The ſequel will make it clear why we begin in this manner.

A	B			
1	—0,000000	A	1,0000000	—0,000000
10	—1,000000	D	3,1622777	—0,500000
100	—2,000000	B	9,0000000	
1000	—3,000000			
10000	—4,000000	C	10,0000000	—1,000000
100000	—5,000000			

2. We ſeek the logarithms of the numbers between 1 and 10, beginning with finding the log. of 9, thus:

Add to each of the three numbers 1, 9, 10, ſeven cyphers, to make the numbers A, B, C, which have the ſame ratio as 1, 9, and 10, and conſequently muſt have the ſame logarithm; therefore multiplying the number A, by the number C, and extracting the ſquare root of the product, will produce the number D, a mean proportional between A and C, (by 16. 6.) and conſequently to have its log. half the difference between the log. of the number A, and that of the number C, muſt be taken, *viz.* 500000, which being added to the log. of the number A, will make 0,500000, the log. of the number D. In the ſame manner may be found the logs. of all numbers, which will be mean proportionals between two others, of which the logs. are already known.

Thus multiplying the number D by the number C, and extracting the root of the product, you will find the mean proportional between D and C, and its log. will be 0,750000. The ſame work is continued in ſearching the mean proportionals between two numbers ſtill nearer to B, till at laſt the number B becomes itſelf a mean proportional between two numbers whoſe logarithms are known, and conſequently its own logarithm will be known, to wit, 0,954242, which will be alſo the logarithm of the number 9.

3. In the ſame manner we find the logarithm of 7, and of 2. Then half the logarithm of 9, gives the log. of its ſquare root 3; the double of the log. of 2, will give the log. of its ſquare 4, ſubtracting the log. of 2, from the log. of 10, gives the log. of 5, adding the log. of 2, to the log. of 3, you have the log. of 6; and adding the log. of 2, to that of 4, you will find the log. of 8.

4. In the ſame manner we find the logarithms anſwering to the numbers between 10 and 100, by making uſe of the logs. of numbers already known, to diſcover thoſe of their products and

roots

roots; and for the reſt, we do as for the numbers 9, 7, and 2.

Note 1. The Tables of Logarithms are uſually carried on to 10000.

2. When we ſeek mean proportionals, we often meet with numbers which have no exact root, and yet we neglect the fractions; ſo that thoſe logarithms are not perfectly exact; but when the root is extracted to many places, ſuch remainders are very inconſiderable, ſo there can ariſe no ſenſible error in their application.

3. The logarithms of all the numbers between 1 and 10, begin with 0; of thoſe between 10 and 100, with 1; of thoſe between 100 and 1000, with 2, *&c.* This firſt figure of a logarithm is uſually ſeparated from thoſe following by a point annex'd, and is call'd the Index, or Characteriſtic, becauſe it ſhews how many places the correſponding number (when integral) conſiſts of, which is always one more than the units in the Index.

Note, *The logarithm of a fraction is the difference of the logarithms of both terms; therefore ſubtract the logarithm of the denominator from the logarithm of the numerator, and the remainder is the logarithm of the fraction, and the number found in the Table correſponding thereto, is the decimal fraction of the ſame value as the vulgar fraction given; but the index of the logarithm of a proper fraction is negative, and has a ſhort line — for minus put over it, and tells the place that the firſt ſignificant figure of the number is below the unit's place; ſo that the index of a logarithm, whether negative or affirmative, is only to ſhew the place the firſt ſignificant of the number poſſeſſes from the unit's place, either below or above it, and is no part of the logarithm to be ſought for in the tables, where we uſe only the fraction, or part to the right hand of the point, which is always affirmative, as Mr.* Gardiner *in page* 1. *of the Explication to his Tables makes evident.*

PROP.

PROP. X.

The Conſtruction of Logarithms for the Sines.

1. The ſine total has for its log 100000000, or perhaps 100000000000; but in moſt tables the laſt three figures are neglected.

2. The ſine of 89 degrees 59′. is found to be 9999999 . . . to find its log. we ſeek in the ordinary tables the log. of 9999. which is 3,9999566, to which we add the log. of 1000000, which is 6,0000000, and we have 9,9999566, the log. of 9999000 . . . Then ſubtracting 9999000 . . . from the ſine total 10000000 . . . the remainder will be found 1000 . . . In the ſame manner ſubtracting the log. 9,9999566 from the log. 10,0000000, the remainder will be 434, by which we may obſerve, that 1000 . . . the difference of the numbers, gives 434 for the difference of the logarithms. Afterwards we ſeek the difference between the given ſine, and the ſine total; which finding to be 1 . . . we ſay, if 1000 . . . gives 434, then 1 . . . will give $\frac{434}{1000}$: : : which being ſubtracted from the log. of the ſine total, will give 99999999 $\frac{566}{1000}$: : : for the log. of the given ſine.

A	10000000	0000000
B	9999999	0000000
C	1	0000000
D		9999999
E	9999998	0000001
F		9999998
G	9999997	0000003
H		9999997
I	9999996	0000006

3. In the ſame manner may be found all the logarithms of the ſines; but to do it more eaſily, we form a geometrical progreſſion, of which 10000000

and 9999999 are the two firſt terms; after this manner we add ſeven cyphers to the ſine total, to make the number A, and the ſame to the ſine 9999999, for the number B, the difference of theſe two numbers will be C, 1,0000000, or $\frac{1}{10000000}$ of the number A; then ſubtracting ſeven cyphers from the number B, it gives the number D, which will alſo be $\frac{1}{10000000}$ of the number B; conſequently if you ſubtract it from the number B, you will have the number E, which will have the ſame ratio to the number B, as the number B to the number A; for as E will be $\frac{9999999}{10000000}$ of the number B, ſo B will be $\frac{9999999}{10000000}$ of the number A. The ſame muſt be done for the number G, I, *&c.* to 100 terms, aſſigning them logarithms, which ſhall diminiſh by the ſame difference, as is between the log. of the firſt number, and the log. of the ſecond, *viz.* $\frac{434}{1000}$, and you will find among thoſe numbers, 15 ſines, with their logarithms.

Further, you muſt raiſe by the ſame way a progreſſion of 50 terms, of which the firſt muſt be the ſine total 10000000, and the ſecond 9999900, the laſt term of the foregoing progreſſion, and you will find ſeveral other ſines with their logarithms.

Laſtly, you muſt make ſeveral other progreſſions, having the ſine total for their firſt term, and the laſt term of the preceding progreſſion for their ſecond, of which the laſt figures will be cyphers.

N. B. You muſt remember to end the progreſſions when their terms will not increaſe enough to reach the ſine which follows. By this means you may find the log. of moſt of the ſines. For the others, they muſt be found after the manner of that of 9999999.

PROP.

PROP. XI.

The Conſtruction of the Logarithms of the Tangents, and Secants.

1. Having found the logarithms of all the ſines, thoſe of the tangents may be eaſily diſcovered, ſince the ſine complement is to the right ſine, as the ſine total to the tangent, (by *Trig. Prop.* 7.) for if the log. of the ſine total be added to the log. of the right ſine, and from the ſum the log. of the ſine complement be ſubtracted, the remainder will be the log. of the tangent.

2. For the ſame reaſon (as above) if from double the log. of the ſine total, the log. of the ſine complement be ſubtracted, there will remain the log. of the ſecant.

Note 1. The logarithms of fractions are nothing but the logarithms of both terms.

2. If in your operations you meet with a log. not to be found in the tables, look out for one the neareſt you can find to it, without regarding the index, or characteriſtic; then adding as many cyphers to the number anſwering to it, as there are units in the index of the log. given, more than in the index of that found, and you will have the number of the log. given to a ſmall fraction, which may be alſo found by this analogy; if the difference between the log. found, and that which follows it in the tables gives 1, the difference between the log. given and that found (without regard to the index) ſhall give the fraction, which, as the log. given is greater or leſs than that found, muſt be accordingly added, or ſubtracted, without regarding the index, or characteriſtic; The Demonſtration of which is eaſy.

PART III.

PLAIN TRIGONOMETRY: *Or, The Reſolution of right-lin'd Triangles.*

SUPPOSITIONS.

1. ALL Triangles are reſolved by the rule of analogy, or proportion, in which three known numbers are given; for multiplying the two laſt, and dividing the product by the firſt, the quotient gives the fourth proportional.

2. We call the ſine, tangent, or ſecant of an angle, the ſine, tangent, or ſecant of the arch which meaſures that angle.

3. In every right-lin'd triangle, as A B C, the half of the ſides are the ſines of the angles to which they are oppoſite: For having deſcribed a circle thro' the three points A B C, it will appear that the half of the ſide B C, for example, is the ſine of half the arch B C, (by *Prop.* 5. *Trig.*) which meaſures the oppoſite angle A, (by 20. 3. *Euc.*)

Fig. 1.

COROLLARY.

From whence it follows, that in every triangle, as A B C, the ſine of the angle A is to its oppoſite ſide B C, as the ſine of any other angle, as B, to its oppoſite ſide A. C. *Fig.* 2.

4. In every triangle, as A B C, rectangular at the point B, if one leg, as A B, be taken for the radius of a circle, or ſine total, the other leg B C, will be the tangent of the angle A, and A C its ſecant. *Fig.* 2.

5. To reſolve a Triangle, ſo much ought to be known of the angles and ſides, as may determine the triangle in ſuch manner, that it cannot be changed, without changing one of the angles or ſides known.

PROP. XII.

The Reſolution of rectangled Triangles.

1. In the triangle A B C, of which the angle B is right, if the line A C be known, and the acute angle C, then we may ſay, by *Suppoſ.* 3. *Fig.* 3.

As the ſine total, or ſine of the angle B : *to the line* A C : : *the ſine of the angle* C : *to the line* A B.

You muſt therefore multiply the line A C by the ſine C, and divide the product by the ſine total, the quotient will be the line A B.

Or by Logarithms. Add the log. of the line A C to the log. of the ſine C, and from the ſum ſubtract the log. of the ſine total; the remainder will be the log. of the line A B. See the work both ways, making C=40° 6′ 26″.

Nat., Sine C - - -	64422	Log. Sine of C - - - - - -	9.8090342
Line A C — —	50	Log. of A C — —	1.6989700
Product - -	3221100	Sum - - - -	11.5080042
Sine total - -	.100000	Log. of Sine total - - -	10.0000000
Quotient A B	32.2	Log. of 32.211 for AB - -	1.5080042

In the ſame manner for the ſide B C, we ſay, by *Suppoſit.* 3.

Sine total: A C : : *ſine* A : B C.

2. In the rectangle triangle A B C, of which the angle B is right, and the angle C known, with the ſide B A oppoſite to it; to find C A, we ſay, by *Sup.* 3.

Fig. 4.

Sine of the angle C : *ſide* A B : *ſine of the angle* B : C A.

And to find the ſide B C, we ſay,

Sine C : A B : : *ſine complement* C : B C.

3. In the triangle A B C, of which the angle B is right, and of which the ſide C B, with the baſe A C, are known, to find the angle A, we ſay,

Fig. 5.

The baſe A C : *ſine of the angle* B : : *ſide*, B C : *ſine of the angle* A.

And to find the ſide A B, we ſay,

The ſine total B : *the baſe* A C : : *ſine complement* A : *ſide* A B.

4. In the triangle A B C, of which the angle B is right, if the two ſides are known; the angle A may be found by ſaying, by *Suppoſit*. 4. *Fig.* 6.

The ſide A B: *the ſine total*: : *the ſide* B C: *tangent of the angle* A.

PROP. XIII.

The Reſolution of oblique-angled Triangles.

1. In the triangle A B C, if the ſide A C be known, and the angles C and B, the ſide A B may be found, by ſaying, by *Suppoſ*. 3. *Fig.* 7.

The ſine of the angle B: *ſide* A C: : *ſine of the angle* C: *ſide* A B.

And becauſe the angle A, is the ſupplement of the two others, to find the ſide B C, we ſay,

The ſine of the angle B: *ſide* A C: : *ſine of the angle* A: *ſide* C B.

2. In the triangle A B C, if the ſides A B, B C, be known, and the angle A, which is not between the known ſides, the triangle cannot be reſolved, unleſs it be known whether the angle C be acute, or obtuſe; becauſe without altering the data, we may change the triangle A B C, into the triangle A B D; but if it be known that the angle C is acute, or D obtuſe, then may the triangle be reſolved by *Sup.* 3. by ſaying, *Fig.* 8.

Side B C: *ſine* A: : *the ſide* B A: *ſine* C, *or* D.

And to find the ſide C A, we ſay,

Sine A: B C: : *ſine* B: A C.

3. In

Fig. 9.

3. In the iſoſceles triangle A B C, if the legs are known, and the angle B between the equal legs, the two other angles which are equal to each other, may be alſo known, and the whole triangle reſolved.

Fig. 10.

4. But in the ſcalenous triangle A B C, if the legs B A, B C, are known, and the angle B, we may know the ſum of the angles A and C; and to find the value of each, uſe the following analogy.

The ſum of the legs A B, B C : *their difference* : : *the tangent of half the ſum of the angles* A *and* C: *to the tangent of an angle, which being aded to the ſaid half, will give the greater angle.*

From the point B deſcribe a circle, whoſe radius let be the leſſer leg B C, which will cut the greater leg in the point D; produce D B till it meets with the circumference of the circle, at the point E; ſo ſhall A E be the ſum of the ſides A B, B C, and D A their difference; the angle D C E will be right, as will alſo the angle C D F, if D F be drawn parallel to C E.

Demonſt. The angle B being common to the two triangles B D C, B A C, the ſum of the angles on the baſe D C, will be equal to the ſum of the angles on the baſe A C; and as the angles on the baſe D C are equal, (by 5. 1 *Euc.*) each of them will be the half of the ſum of the angles A and C; From whence it may be concluded, 1. That the line C E muſt be the tangent of half the ſum of the angles A and C. 2. That the angle A C D, of which D F will be the tangent, being added to the half of the ſum of the angles A and C, will be equal to the angle C, and conſequently, ſince (by 4. 6. *Euc.*)

A E is to D A, as C E, to D F, it is certain, that the ſum of the ſides B A, B C, is to their difference as the tangent of half the ſum of the angles A and C, is to the tangent of an angle, which added to the ſaid half, will make the greater angle C.

5. If all the ſides of the triangle A B C are known; from the greateſt angle B, let fall the perpendicular B D, upon the greateſt ſide A C, and it will form two rectangle triangles, the angles whereof may be reſolved by the foregoing propoſitions; but to find D the point where the perpendicular falleth in the line A C, we uſe the following analogy. *Fig.* 11.

The greateſt ſide A C: *the ſum of the other two ſides* B A, B C: : *the difference of the ſides* B A, B C, *viz.* G A: A F, which ſubtracted from A C, the point D is in the middle of the remainder.

It is eaſy to ſee how the figure is made.

Demonſt. The line A B E, is the ſum of the ſides A B, B C, and the rectangle E A G is equal to the rectangle C A F, (by *Cor.* 16. 6. *Eucl.*) therefore C A: the ſum of the ſides A B, B C: : A G: A F, (by 14. 6. *Eucl.*)

REMARK.

If in the figure A B C D, the angles B A C, BAD, BDA, BDC, be known, which are ſufficient to determine the other angles, all the other angles may be known, without knowing the ſide A B; to which you may give what value you pleaſe, without changing the angles ſought. Thus knowing any one ſide, the whole figure may be reſolved. *Fig.* 12.

PART

PART IV.

The Reſolution of SPHERICAL TRIANGLES.

LEMMA I.

Fig. 1. IF a Globe or Sphere is cut by a plane paſſing thro' its center A, the common ſection will be a circle, whoſe radius A B, will be the radius of the Globe, and its circumference will encompaſs the ſurface of the Globe; ſuch circle is call'd a *Great Circle.*

LEMMA II.

Fig. 2. If a Sphere is cut by the plane E F G H, not paſſing thro' the center A, the common ſection will be a circle, the radii of which will be leſs than thoſe of the Sphere. Draw A B perpendicular to the plane, and B M, B N, B L, to the ſurface of the Globe, and A M, A N, A L, and you will have three triangles equal in every reſpect, of which the ſides B M, B N, B L, will be equal to one another, and leſs than A L, A M, A N, radii of the Sphere; ſuch circle is call'd a *Leſſer Circle.*

DEFI-

DEFINITIONS.

1. A Spherical Angle is form'd by two arches of a great circle, as A B C, *Fig.* 3.

2. A Spherical Triangle is a part of the ſurface of a Sphere, bounded with three arches of great circles, as A B C. *v. Fig.* 3.

3. The Pole of a Circle is a point in the ſurface of a ſphere, equidiſtant from every part of the circumference of the ſaid circle. Thus the points A and B are the poles of the circles C D and E F. *Fig.* 4.

SUPPOSITIONS.

1. All ſpherical triangles are ſuppoſed to be nade upon the ſurface of a ſphere, of which the adius contains 10000 . . . equal parts.

2. If in a ſphere two great circles A B C, A D C, ut each other in the points A C, their ommon ſection will paſs thro' the cen- er of the ſphere, which is common to oth, and will conſequently be the diameter of oth, and will divide both equally. *Fig.* 5.

3. The line A B, which joins the two poles of e circle C D, is the diameter of the here. For imagining a great circle o paſs thro' the points A and B, which ill cut the circle C I D in the points C D, the rches A C, and A D, B C, and B D, will be ual; ſince therefore the whole arch A C B, will be *Fig.* 6.

be equal to the whole arch A D B, the line A B will be the diameter of the great circle C B D A, and conſequently of the ſphere.

Note, If C I D is a great circle, C B and C A will be quadrants; as will all other arches of a great circle drawn thro' the poles A, B, upon the circle C I D.

Fig. 7.

4. Every ſpherical angle, as A B C, is meaſured by the arch A C of a circle, of which B is the pole. This definition ſuppoſes, that the arches of all circles whoſe pole is B, will be ſimilar to the arch A C, if they are between the arches B A, B C. From whence it follows, that the angles diametrically oppoſite B D are equal, being meaſured by the ſame arch A C; ſince by the foregoing Suppoſition, the point D will be alſo the pole of the arch A C.

Fig. 7.

5. The arch B C falling upon the arch A E makes two angles A C B, E C B, equal to two right, ſince they are meaſured by the ſemicircle A B E, (by *Sup*. 2.) In the ſame manner the oppoſite angles B C E, A C D, will be equal, being each the ſupplement of the angle A C B.

Fig. 8.

6. If the circle A B C, paſſes thro' the pole A of the circle B E D, they will cut each other at right angles. For deſcribing a great circle, of which let B be the pole, it will paſs thro' the points A C; ſo that B A, B C, will be quadrants of a circle, (by *Sup*. 3.) but A E will alſo be a quadrant as well as E C.

Therefore the angles A B E, C B E, will be meaſured by quadrants.

7. If the circle B A D, makes right angles with the cirle B E D, they will alſo paſs thro' each other's pole. *Fig.* 8.

8. If the point B, be not the pole of the circle D F E, but ſome other point, as A, the arch B A E will be greater than any other arch, as B F. *Fig.* 9.

Draw B C right upon the plane D E F, and it will fall upon D E, (by 38. 11.) at the point C, which will not be the center of the circle D F E; otherwiſe the lines B D, B E, would be equal. Draw B F, C F.

Demonſt. The line C E paſſing thro' the center of the circle, will be greater than C F, (by 7. 3.) Then in the rectangled triangles E C B, F C B, which have the ſide C B common, the ſide C E, being greater than the ſide C F, the baſe B E will be greater than the baſe B F; and becauſe arches increaſe in proportion with their cords, till they become a ſemicircle, the arch B A E will be greater than B F.

9. The Figure remaining as it was, the arch B D will be leſs than any other, as B F, and B F will be greater than any other which can be made between D and F, and leſs than any other between F and E.

10. In every triangle, as A B C, each ſide, as A B, is leſs than a ſemicircle. *Fig.* 11.
For their arches A B, A C, being prolonged till they

they meet in the point D, the arch A D will be but a ſemicircle, (by *Sup.* 2.)

11. In every triangle, as A B C, two ſides A B, A C, are greater than the third B C.

Fig. 11.

From the point B, with the diſtance B A, deſcribe the circle D A F, and compleat the circle B C G F.

Demonſt. Since the point B is one pole of the circle A D E F, the point C, which is diſtant from it leſs than the ſpace of a ſemicircle, (by the preceding) cannot be its other pole, (by *Sup.* 3.) but ſome other point, as G, if B G is a ſemicircle; then the arch C A is greater than C D, (by *Sup.* 9.) But B A and B D are equal, (by *Def.* 3.) therefore C A, and B A, are greater than C B.

12. The three ſides of a ſpherical triangle A B C, are leſs than two ſemicircles.

Fig. 12.

For continuing A C, A B, till they meet in the point D; their arches A C D, A B D, are but equal to two ſemicircles: but C D, B D, are greater than C B, (by the preceding); therefore A C, A B, C B, are not equal to two ſemicircles.

13. If the triangle A B C, has two legs B A, B C, equal to the two legs of another triangle E F, F G; and if the angle B, be equal to the angle F; the two triangles are equal in every reſpect.

Fig. 13.

Demonſt. If one triangle be laid on the other, A B will cover F E, and B C will cover F G: therefore the baſe A C will cover the baſe E G; otherwiſe they would cut each other in the points E, G, E, G, and each baſe would be a ſemicircle, (by *Sup.* 2) which cannot be (by *Sup.* 10.)

14. It is demonſtrable alſo in ſpherical triangles, as in rectilinear, that if the legs A B, A C, are equal to the legs E D, D F; the angle included will be greater, or leſs, in the one triangle than in the other, according as their baſes are greater or leſs. In the ſame manner it is demonſtrable, that iſoſceles triangles have equal angles on their baſes, and that thoſe which have equal angles on their baſes, are iſoſceles triangles. *Fig.* 14.

15. If in the triangle A B C, the angle A is greater than the angle C; the ſide C B will be greater than the ſide A B. *Fig.* 15.

Demonſt. Make the angle D A C, equal to the angle D C A, and the arch A D will be equal to the arch D C, (by the preceding) but D B, D A, are greater than A B, (by *Sup.* 13.) therefore B D, D C, will be greater than A B.

16. In the triangle A B C, whoſe angle A is right, the angles C and B have the ſame affection as the ſides oppoſite to them; that is to ſay, they are right, acute, or obtuſe, according as the oppoſite arches are equal to, or leſs, or greater than a quadrant. *Fig.* 16.

Demonſt. Since the arch A B is perpendicular upon the arch A C, it will paſs thro' its pole, (by *Sup.* 7.) therefore if the arch A B is a quadrant, the point B will be the pole of the arch A C (by *Sup.* 3.) and conſequently the angle C will be right (by *Sup.* 6.) but the arch A B cannot be aug-

mented

mented, or diminiſhed, without augmenting or diminiſhing the angle A C B: Therefore the angle C will be right, or greater, or leſs than a right, according as the arch A B is a quadrant, or greater, or leſs than a quadrant.

17. If the two ſemicircles A B C, A D C, cut each other at right angles; the point B, *Fig.* 17. the pole of the ſemicircle A D C, will divide A B C into two quadrants, (by *Def.* 2.) In the ſame manner the point D, the pole of the ſemicircle A B C, will divide A D C into two quadrants. Farther, the arch D E will be a quadrant, (by *Sup.* 3.) and ſince E is not the pole of the arch A D C, the arch E D will be greater than the arch E F, and leſs than the arch E G, (by *Sup.* 9)

COROLLARIES.

1. In every rectangled triangle A D E, the baſe D E will be a quadrant, if at leaſt the *Fig.* 17. leg A D is the fourth part of a circle: But if the two legs are greater than a quadrant, as in the triangle C E F, or if they are both leſs than a quadrant, as in the triangle A F E, the baſe E F will be leſs than a quadrant. Laſtly, if one of the legs is leſs than a quadrant, and the other greater, as in the triangle C E G, the baſe E G will be greater than a quadrant.

2. The three ſides of a rectangled triangle cannot each of them be greater than a quadrant; nor can one ſide be greater than a quadrant, and the other two leſs.

3. The baſe of a rectangle triangle, or the arch oppoſite to the right angle, cannot be leſs than a quadrant, without affecting the legs, and conſequently the angles.

Prop. I.

If the ſpherical triangle A B C, having every one of its ſides leſs than a quadrant, has the angle B right; the ſine of the angle B will be to the ſine of the oppoſite arch A C, as the ſine of another angle A, to the ſine of the oppoſite arch B C. *Fig.* 18.

Continue B C, till B F becomes a quadrant, ſo ſhall the point F be the pole of the arch A B, which muſt alſo be continued till A E becomes a quadrant: then deſcribing the arch F E, of which the pole muſt be A, and conſequently the arch A C L will be a quadrant, and L E the meaſure of the angle A. If you draw L H right upon the circle A B E, it will fall perpendicularly upon the common ſection G E, and will be the ſine of the arch L E, or of the angle A: L G will be the ſine total, or ſine of the angle B.

Farther, draw thro' the point C, a plane parallel to the circle E L F, and like it perpendicular upon the plane A B E. This plane will cut the plane, B C F, perpendicular upon A B E, and their common ſection C D will be right upon the ſame plane, and conſequently parallel to the line H L, and the ſine of the arch B C. In the ſame manner I D, and G H, C I, and G L, are parallel, (by 16. 11,) being the common ſections of the two parallel planes with a third; and as L G is found perpendicular upon G A, I C will be alſo perpendicular upon G A, and the ſine of the arch C A.

Demonſtrat. The three ſides of the triangle L H G, being parallel to the three ſides of the triangle C I D, the two triangles are equiangular, (by 10. 11.) therefore (by 4. 6.) L G, the ſine of the angle B, will be to C I, the ſine of the oppoſite arch A C, as L H, the ſine of the angle A, to C D, the ſine of the oppoſite arch B C.

Lemma.

If the plane A, is perpendicular to the plane B, and C D perpendicular upon the common ſection E H, it will be right upon the plane; ſince the plane A is a web of right lines, upon the plane B. *Fig.* 19.

Prop. II.

In the ſame triangle A B C, the tangent of the angle A, will be to the tangent of the oppoſite leg C B, as the ſine total to the ſine of the other leg A B. *Fig.* 19.

Continue the arches A B, A C, B C, as above; then draw E I, the tangent of the arch D E, or of the angle A; and B D, the tangent of the arch B C; and by the foregoing *Lemma* they ſhall be right upon the plane A B E.

Farther, thro' the line B D, draw a plane parallel to the plane E L F, and like it perpendicular upon the plane A B E; the common ſections D M, B M, will be parallel to the common ſections I G, E G; and as E G is perpendicular upon F G, B M will be alſo perpendicular upon F G; and will conſequently be the ſine of the arch B A.

Demonſt. Since all the ſides of the triangle B D M, are parallel to the ſides of the triangle I E G; the two triangles are equiangular, (by 10. 11.) therefore E I, tangent of the angle A, will be to B D, tangent of the arch B C, as E G, the ſine total to B M, the ſine of the leg B A.

COROLLARY I.

By theſe two Propoſitions, are all Caſes of right-angled triangles reſolved, in the manner following; provided all their arches are leſs than quadrants.

Let A B C be a triangle, whoſe angle C is right; continue C A to G, ſo that G C may be a quadrant, and G the pole of the arch B C, (by *Sup.* 3.) continue alſo B C, ſo that B I may be a quadrant, which will likewiſe make the arch G I a quadrant, (by *Sup.* 3.) and its pole B, and conſequently B A being continued to the point H, will alſo be a quadrant. In the ſame manner continue C B, ſo that C D may be a quadrant, and A C, till A F becomes a quadrant; for then D F and A E will be alſo quadrants,

Fig. 20.

CASE I.

If the angle A, and the oppoſite leg B C, be known; the baſe A B may be found, by ſaying, by the firſt Propoſition,

Sine of the angle A : *ſine of the leg* B C : : *ſine total* : *ſine of the baſe* A B.

Or the leg A C may be found, by ſaying, by the ſecond, *Tangent of the angle* A : *tangent of the arch* B C : : *ſine total : ſine of the leg* A C.

Sine total : *tangent of* B C : : *Co-tangent of* A : *ſine of* A C,

The angle B may be alſo found, becauſe the triangle B E D will have its baſe B D, the complement of B C, which is known ; and its leg D E, the complement of E F, which may be known, becauſe it is the meaſure of the known angle A. We ſay therefore, by the firſt,

Sine complement of B C : *ſine total* : : *ſine complement of the angle* A : *ſine of the angle* B.

CASE II.

If the angle A be known, and the adjacent leg A C, the other leg B C may be known, by the ſecond Propoſition, thus,

Sine total : *ſine of the leg* A C : : *tangent of the angle* A : *tangent of the arch* B C.

Alſo the baſe A B may be found, by the ſecond, thus,

Sine total : *ſine* E D, *complement of the angle* A : : *tangent of the angle* D, *complement of the arch* A C : *tangent of the arch* E B, *complement of the baſe* A B.

Or the angle B may be found, by the firſt, thus,

Sine total H : *ſine of the baſe* A G, *complement of* A C : : *ſine of the angle* A : *ſine of the arch* G H, *complement of the angle* B.

CASE III.

If the angle A, and the baſe A B be known, the leg B C may be found, by the firſt, thus,

Sine total : *ſine of the baſe* A B : : *ſine of* A : *ſine of the leg* B C.

Or the other leg A C may be found, by the ſecond, thus,

Sine D E, *complement of the angle* A : *ſine total* :: *tangent* B E, *complement of* A B : *tangent of angle* D, *complement of* A C.

Or thus,

Sine total : *ſine* D E, *complement of the angle* A :: *tangent* A B, *complement of* B E : *tangent* A C, *complement of the angle* D.

And the angle B may be known by the ſecond. thus,

Sine total ſine A H, *complement of* A B :: *tangent of the angle* A : *tangent* H G, *complement of the angle* B.

CASE IV.

If the legs A C and B C are known, the baſe A B may be found, by the firſt, thus,

Sine total H : *ſine* G A, *complement of* A C :: *ſine* G, *complement of* B C : *ſine* H A, *complement of* A B.

The angle A may alſo be found by the ſecond, thus,

Sine of the arch A C : *ſine total* :: *tangent of the arch* B C : *tangent of the angle* A.

In the ſame manner may be found the angle B.

Sine of the arch B C : *ſine total* :: *tangent of* A C : *tangent of the angle* B.

CASE V.

If the leg A C, and the baſe A B, are known, the other leg B C, may be found by the firſt thus,

Sine of the arch G A, *complement of* A C : *ſine total* H : *ſine* A H, *complement of* A B :: *ſine* G, *complement of* B C.

The angle A may alſo be found by the ſecond, thus,

Tangent of the angle D, *complement of* A C: *tangent of* B E, *complement of* A B:: *ſine total*: *ſine* D E, *complement of angle* A.

And the angle B may be found by the firſt, thus,

Sine of the arch A B: *ſine total* C:: *ſine* A C: *ſine angle* B.

CASE VI.

If all the angles are known, the baſe A B may be found by the ſecond, thus,

Tangent of the angle A: *tangent of the arch* H G, *complement of the angle* B:: *ſine total*: *ſine* H A, *complement of the baſe* A B.

Or the leg A C may be found by the firſt, thus

Sine of the angle A: *ſine of the arch* H G, *comp. angle* B: *ſine total*: *ſine* A G, *complement of* A C.

In the ſame manner may be found the leg B C.

COROLLARY II.

Fig. 21.

Such rectangle triangles as have two ſides greater than quadrants, are reſolv'd as follows. Let the triangle A B C be propoſed, whoſe two ſides A B, B C, are greater than quadrants; produce the ſaid two ſides till they meet at the point D, which will give another triangle C A D, whoſe angle D will be equal to the angle B, and the ſide A C common to both; the other angles, and the other ſides, will be the ſupplements to the other angles and ſides of the triangle A B C; ſo that it will be eaſy to reſolve the triangle A B C by the triangle A C D.

If the rectangled triangle E F G, has two of its ſides E F, E G, equal to a quadrant, the angles G and F will be right; and the arch F G will be the meaſure of the angle E, (by *Sup.* 17.) the one or the other of which muſt be known, to reſolve the triangle.

Fig. 22.

Note, Some reſolutions will be equivocal, becauſe the ſine equally reſpects both the arch, and its ſupplement; and in ſuch caſes, the triangle cannot be reſolved, unleſs we know the affection of the angle, or of the arch found, by means of the ſine. Thus the triangle A B C cannot be reſolved, tho' the angle A, and the arches A C and B C are known, unleſs we alſo know the affection of the arch A B, all the reſt agreeing as well with the triangle D C A.

Fig. 23.

Prop. III.

In every oblique-angled triangle, as A B C, the ſine of the angle A, is to the ſine of the oppoſite arch B C, as the ſine of the angle B, to the ſine of the oppoſite arch A C: Draw the perpendicular arch C D, which may fall either within, or without the triangle.

Fig. 24.

Demonſt. Since the triangle A D C is rectangled (by *Prop.* 1.) the ſine total D : to the ſine of A C : : ſine A : ſine of C D, and conſequently, the rectangle under the ſine total, and the ſine C D, is equal to the rectangle under the ſine A, and the ſine of A C, (by 15. 6.) In the ſame manner, the triangle D B C, being rectangled, the ſine total D : ſine C B : : ſine B : ſine C D, from

whence

whence it will again follow, that the rectangle under the ſine total, and the ſine C D, will be equal to the rectangle under the ſine C B, and the ſine B, as it was equal to the rectangle under the ſine A C, and the ſine A; ſince then the rectangle under the ſine A C, and the ſine A, is equal to the rectangle under the ſine C B, and the ſine B; therefore (by 15. 6.) the ſine A : ſine C B : : ſine B : ſine A C.

LEMMA.

If in the triangle A B C, the arch which falls from the angle A, perpendicularly upon *Fig.* 25. the baſe B C, be found within the triangle, as A D; then the angles B and C will have the ſame affection as the arch A D, (by *Sup.* 16.) But if the perpendicular arch falls without the triangle, as A E, then the angle B, and the ſupplement of the angle C, *viz.* A C E, will have the ſame affection as the ſaid arch A E, and ſo the angles B C prove diverſly affected.

COROLLARY.

By this Propoſition therefore oblique-angled triangles may be reſolved, by thus letting fall perpendiculars, and thereby rendering them rectangled triangles. But their reſolution will be rendered more eaſy by the Propoſition following.

PROP. IV.

Let the oblique-angled triangle A B C be propoſed, of which the legs A B, A C are *Fig.* 26. leſs than quadrants: Draw the perpendicular arch A D, which will fall either within, or without the triangle, and produce it till A G becomes a quadrant; then from the point A, as a pole, deſcribe the quadrant G H, ſo that H may be the pole of the arch A G; the arches D B, D C, being prolonged, will paſs thro' the point H, (by *Sup.* 6) ſo that D B H, D C H, will be quadrants, as will alſo A E, and A F, and conſequently C E, B F, will be complements of the arches A C, A B, as B H, C H, will be complements of the arches D B, and D C; and the arches F H, E H, will be complements of the arches G F or G E, or of the angles G A F, G A E. From the figure thus prepared, may be drawn the following Concluſions; but obſerve that the ſame figure muſt be made for the triangle B I C, of which the legs I B and I C are greater than quadrants.

CONCLUSIONS.

1. *The tangents complements of the legs, and the ſines complements of the angles, which the perpendicular arch makes within the legs, are proportional.*

In the rectangle triangle B F H, the tangent of the angle H, is to the ſine total, as the tangent of the arch B F, to the ſine of the arch F H, (by *Prop.* 2.) In the ſame manner in the triangle C E H, as the tangent of the angle H, is to the ſine total; ſo the tangent of the arch C E, is to the ſine of the arch E H. Therefore, as the tangent of the arch C E, the complement of A C, is to the ſine of the arch E H, the complement of the angle E A G; ſo the tangent of the arch B F, the complement of A B, will be to the ſine of the arch F H, the complement of the angle F A G.

2. *The ſines complements of the legs, and the ſines complements of the arches between the legs, and the perpendicular arch, are proportional.*

In the triangle H B F, (by *Prop.* 1,) as the ſine of the angle H, is to the ſine total; ſo the ſine of the arch B F, is to the ſine of the arch B H. In the ſame manner, in the triangle C E H, as the ſine of the angle H, is to the ſine total; ſo the ſine of the arch C E, is to the ſine of the arch C H. Therefore, as the ſine of the arch B F, the complement of the leg A B, is to the ſine of the arch B H, the complement of B D; ſo the ſine of the arch C E, the complement of the leg A C, will be to the ſine of the arch C H; the complement of C D.

3. *The tangents of the angles of the baſe, are reciprocally proportional to the ſines of the arches, between the perpendicular arch, and the legs.*

In the triangle B A D, (by *Prob.* 2.) the ſine total : ſine B D :: tangent B : tangent A D. Therefore the rectangle under the ſine total, and the tangent A D, is equal to the rectangle under the ſine B D, and the tangent B, (by 15. 6.) In the ſame manner, in the triangle A D C, the tangent A D : the tangent C :: ſine D E : ſine total, and conſequently, the rectangle under the ſine total, and the tangent A D, will be alſo equal to the rectangle under the ſine D C, and the tangent C. Therefore the rectangle under the ſine D C, and the tangent C, is equal to the rectangle under the ſine B D, and the tangent B; and therefore (by 14. 6.) the tangent of the angle C : to the tangent of the angle B :: the ſine of the arch B D : the ſine of the arch D C.

4. *The ſines of the angles made by the perpendicular arch. are proportional to the ſines complement of the angles of the baſe.*

In the triangle B A D. (*Caſe* 1. *Cor.* 1. of *Prop.* 1. & 2.) the ſine of the angle B A D, is to the ſine complement of the angle B, as the ſine total is to the ſine complement of the arch A D. In the ſame manner, in the triangle C A D, the ſine of the angle C A B, is to the ſine complement of the angle C, as the ſine total to the ſine of the arch A D; therefore the ſine complement of the angle B, is to the ſine of the angle B A D, as the ſine complement of the angle C, to the ſine of the angle D A C.

COROLLARIES.

1. If in the triangle A B C, the angles A and B, with the ſide A B, be known, the triangle may be reſolved, becauſe the Data are ſufficient to determine it.

Fig. 27

Imagine the arch A D, to fall perpendicnlarly from the angle upon the oppoſite ſide B C, in ſuch manner, that the angle B, which is oppoſite to the arch A D, may be acute; this will produce three different caſes, as they are expreſs'd in the figure and which all have the ſame ſolution.

To find the angle C, look in the rectangled triangle A B D, wherein you know the acute angle B, and the hypothenuſe A B; look, I ſay for the angle B A D, (by *Caſe* 3. of *Cor.* 1. *Prop.* 1. & 2.) afterwards, ſay (by *Concluſion* 4.)

The ſine of the angle B A D : *ſine complement of the angle* B : : *the ſine of the angle* D A C : *to the ſine of the angle* C.

To know the ſide A C, ſay (by *Concluſion* 1.) *Sine complement of the angle* B A D: *ſine complement of the angle* D A C :: *tangent complement of the arch* A B : *tangent complement of the arch* A C.

In the ſame manner we may know B C, by letting the perpendicular arch fall on A C.

COROL. II.

If in the triangle A B C, the angles A and B are known, with the ſide C B, let fall the perpendicular arch C D, as above, which will give you the rectangle triangle B C D, of which you know the angle B, and the baſe B C; therefore you will have the angle B C D, (by *Caſe* 3. of *Cor.* 1. *Prop.* 1. & 2.) Afterwards, to find the angle B C A, ſay (by *Concluſion* 4)

Fig. 28.

The ſine comp. of the angle B : *ſine comp. of the angle* A :: *ſine of the angle* B C D : *ſine of the angle* D C A ; *then the ſum or difference of the two angles will be the angle* B C A.

To find the ſide C A, ſay (by *Prop.* 3.)

The ſine of the angle A : *ſine of the arch* B C :: *ſine of the angle* B : *ſine of the arch* A C.

To find the arch B A, ſeek in the triangle B C D, (by *Caſe* 3. of *Cor.* 1. *Prop.* 1. & 2.) the leg B D ; then (by *Concluſion* 3.) ſay,

The tangent of the angle A : *tangent of the angle* B : *ſine of the arch* B D : *ſine of the arch* A D.

Obſerve,

1. If the arch C D be the ſupplement of the arch C A, and if D C, D B, be continued till they meet in the point E, the arch C D will be alſo the ſupplement of

Fig. 29

of the arch C E; and conſequently the triangle A C E will be an iſoſceles, and the angles A and E equal; as will alſo A, D; therefore the different triangles B C A, B C D, will have the arch C B common, and two angles equal; ſo that to reſolve the triangle A B C, it is not ſufficient to know the angles A, B, with the ſine C B, but the affections of the angles, and of the unknown ſides, muſt be alſo known.

2. If in the rectangled triangle A C D, the baſe A C is grater than a quadrant, the angles A C D, C A D, will be diverſly affected. (by *Sup.* 17.) and on the contrary; from thence it follows, that to determine the affection of the arch A C, and of the angle C A D, it is neceſſary to know the affection either of the one, or the other. The ſame may be ſaid of the arch B A; for if in the rectangle triangle B C A, the baſe C A is greater than a quadrant, the legs B A, B C, will be of different affection, and the contrary.

COROL. III.

If the two ſides A B, A C, of the triangle A B C are known, and the angle A, draw the perpendicular C D from one of the unknown angles, upon one of the known ſides; then to know the angle B, you muſt find the arch A C, (by *Caſe* 3.) and (by *Concluſion* 3.) ſay,

Fig. 30.

Sine B D : *ſine* A D : : *tangent* A : *tangent* B.

The ſame muſt be done for the angle C; then to have the arch C B, you ſay (by *Concluſion* 2)

Sine comp. A D : *ſine comp.* D B : *ſine comp.* A C : *ſine comp.* C B.

COROL.

COROL. IV.

If in the triangle A B C, the ſides A B, A C, and the angle B, are known, the triangle cannot yet be reſolved, without knowing the affection of the angle C; *Fig.* 31. for ſuppoſing the arches A C, A D, are equal, and that the arch D C be prolong'd till it meets with the arch B A in the point B; then the different triangles A B D, A B C, will have two ſides equal and one angle, equal; but then, the angle A C B will be the ſupplement of the angle A D B; ſo that in knowing the affection of the angle C, the triangle may be determined. *Fig.* 32. To know then the angle C, ſay (by *Prop.* 3.)

Sine A C : *ſine of the angle* B : : *ſine of the arch* A B : *ſine of the angle* C.

Farther, to find the angle A, let fall the perpendicular arch A D, as in the firſt Caſe, and look in the rectangled triangle A D B, the angle B A D, (by *Caſe* 3. of *Cor.* 1. *Prop.* 1 & 2.) then ſay (by *Concluſion* 1.)

Tangent complement of the arch B A : *tangent comp. of the arch* A C : : *ſine comp. of the angle* B A D : *ſine comp. of the angle* D A C.

Laſtly, to know the ſide B C, you muſt ſeek in the rectangle triangle A D B. the leg B D, (by *Caſe* 3. of *Cor.* 1 *Prop.* 1. & 2.) then ſay (by *Concluſion* 2.)

The ſine comp. of the leg A B : *ſine comp. of the leg* A C : : *ſine comp. of the arch* B D : *ſine comp. of the arch* D C.

Prop. V.

In every ſpherical triangle, wherein the unequal legs A C, A B, from the angle A, the rectangle contain'd under the ſines of the legs A B, A C, is to the ſquare of the radius, as the difference between the vers'd ſine of the baſe C B, and the vers'd ſine of the difference of the legs, will be to the vers'd ſine of the angle A.

Fig. 33.

Make the circle A B N A, aad the ſemicircles B C N, A C M; then thro' the point C, make a circle of which let B be the pole, and another, whoſe pole let be A; theſe two circles will appear in the figure, as the right lines F S, I P: Further, draw the perpendiculars L O upon N B, L H upon A M, H K upon O L, F T upon B N, and F G npon I P.

Explanation of the Figure.

Since the arches A C, A F, falling from the pole A, upon the Circle F S, are equal, F B will be the difference of the legs A C, A B, and its vers'd ſine ſhall be T B. In the ſame manner, the baſe B C is equal to the arch B I, of which D B is the vers'd ſine; and conſequently D T, or F G, will be the difference betwcen the vers'd ſine of the baſe B C, and the vers'd ſine of the difference of the legs F B. Farther, A B H, and B H O, being quadrants, the arches A B, H O, are equal; and H K is the ſine of the leg A B, as F E is the ſine of the leg A C, equal to the arch A F. Laſtly, C F is the vers'd ſine of the arch C F ſimilar to the arch which meaſures the angle

A,

A, (by *Def.* 4.) Let us ſuppoſe that H V is equal to the vers'd ſine of the ſaid angle; it is then to be demonſtrated, that T D, or its equal F G: H V : : the rectangle under thee ſines H K, F E: the ſquare of the radius H L.

Demonſtrat. The triangle H K L, is ſimilar to the triangle F C G, ſince they are both rectangled, and their angles L and C are equal, being inward alternates of a third L G C; therefore (by 4. 6.) K H: H L: : F G: F C. Farther, ſince F C and H V are the vers'd ſines of two ſimilar arches, F C: H V : : F E radius of F C: H L radius of H V. But the ratio of the rectangle E F H K, to the ſquare of L H, is compoſed of the ratio of H K to H L, and of the ratio of F E to H L, (by 23. 6.) therefore the ſame ratio will be alſo compoſed of the ratio of F G to F C, and of the ratio of F C to H V; and as the ratio of F G to H V, is alſo compoſed of the ratio of F G to F C, and of the ratio of F C to H V; the ratio of the abovesaid rectangle, to the aboveſaid ſquare, will be equal to the ratio of F G to H V, as was to be demonſtrated.

Fig. 33.

COROLLARY.

Fig. 33. This Propoſition gives us the following analogy, for finding the angle A.

H K *the ſine of one leg* : H L *the ſine total* : : F G *the difference between the vers'd ſine of the baſe* B C, *and the vers'd ſine of the difference of the legs* : F C *a fourth ſine.* Then, F E *the ſine of the other leg* : H L *the ſine total* : : F C *the fourth ſine found* : H V *the vers'd ſine of the angle* A.

LEMMA I.

If A B is the radius of a circle, and B E the ine of half the arches B D, D C; if, rther, we take C M, the difference of he arches B D, D C; I ſay, that the ine total ſhall be to B E, the ſine of half the arches BD, D C, as C H, the ſine of half their difference, o C I, the half of C L, or G N, which is the difference between the vers'd ſine of the great Arch BD, and the vers'd ſine of the leſſer one D C. *Fig.* 34.

Demonſtrat. Since the angle E A B, which is meaſured by half the arch B C, is equal (by 20. 3.) o the angle C M B or I H C, for C I being the alf of C L, and C H the half of C M, I H will e parallel to M L (by 2. 6.) the two rectangles A B E, I H C, will be ſimilar; and conſequently (by 4. 6.) A B : B E : : C H : C I. From whence e conclude, that the rectangle contain'd under he ſine of the half of two arches, and the ſine of alf their difference, will be always equal to the ectangle contained under the ſine total, and half he difference of the vers'd ſines of the ſame rches.

LEMMA II.

The ſquare of A C, which is the ſine of half he arch A B, is equal to the rectangle nder half the radius, and under the ers'd ſine D B. Draw the perpendiulars A D, E C. *Fig.* 35.

Demonſtrat. The rectangled triangles A D B, E C B, having the angle B common, will be ſimilar; therefore (by 4. 6.) E B : B C :: B A : B D, or the half of E B : B C :: B C half of A B : B D, (by 16. 6.) &c.

Prop. VI.

Fig. 26.

In every triangle, as A B C, the rectangle under the ſines of the arches B A and C A, is to the ſquare of the ſine total, as the rectangle under the ſine of half an arch compoſed of the baſe B C, and of the difference of the arches B A and A C; and under the ſine of half the difference between the baſe, and the difference of the arches A B and A C, to the ſquare of the ſine of half the angle A.

Demonſtrat. By the foregoing, the rectangle under the ſines of the arches A B and A C, is to the ſquare of the ſine total, as the difference between the vers'd ſine of the baſe B C, and the vers'd ſine of the difference of the arches A B and A C, to the vers'd ſine of the angle A. Put in the place of theſe two laſt lines, two rectangles, which ſhall have the ſine total for their baſe, and theſe two lines for their height, or the half of theſe rectangles, and you will find that the rectangle under the ſines of the arches A B and A C, will be to the ſquare of the ſine total, as the rectangle under the ſine total, and under half the difference between the vers'd ſine of the baſe B C, and the vers'd ſine of the difference of the arches A B and A C, is to the rectangle under half the ſine total, and the vers'd ſine of the angle A. But by *Lemma* 1. this third rectangle is equal to the rectangle under the ſine of half an arch compoſed of the baſe B C, and the difference of the arches A B and A C, and under the ſine of half the difference between the baſe B C, and the difference of the arches A B and A C; and by *Lemma* 2 the fourth rectangle is equal to the ſquare of the ſine of half the angle A. Therefore the rectangle under the ſine of the arches A B and A C, &c.

COROLLARY.

To find the angle A of the triangle A B C, of which all the ſides are known.

1. *Add the logarithms which anſwer to the ſines of the two arches* A B, A C.

2. *Double the logarithm of the ſine total.*

3. *Add the logarithm which anſwers to the ſine of half an arch compoſed of the baſe* B C, *and of the difference of the arches* A B, A C, *to the logarithm which anſwers to the ſine of half the difference between the baſe, and the difference of the arches* A B, A C.

Then will you have three terms, of which the firſt being ſubſtracted from the ſum of the two others, half the remainder will be the logarithm anſwering to the ſine of half the angle A.

Note, If the ſides A C, A B, were equal, we ſhould draw a perpendicular upon the baſe, which would divide it equally.

PROP. the Laſt.

Let the triangle A B C be propoſed, wherei
A is the greateſt angle, and is alſo th
Fig. 37. pole of the circle K F D; let B be th
pole of the circle H E D, and C th
pole of the circle I E. Theſe three circles wi
form the triangle F E D, of which the ſide F
oppoſite to the angle A, will be its ſupplemen
the ſide F E will be the meaſure of the angle
and the arch D E the meaſure of the angle
Compleat the circle A C, and all the other arche
till they terminate in the ſaid circle.

Demonſtrat. Since the points A, B, C, are th
poles of the circles K D, H D, I E, the angl
I, K, L, M, G, H, will be right, (by *Sup.* 8.) an
for the ſame reaſon the points D, E, F, will b
the poles of the arches A B, B C, A C; fro
whence it follows, that D G will be a quadran
(by *Sup.* 4.) as F K; and G K, which meaſur
the ſupplement of the angle A, will be equal
F D. In the ſame manner E L, and F I, will
quadrants, and conſequently I L, which meaſu
the angle C, will be equal to the arch F E. Laſtl
D H, and E M, will be quadrants; and con
quently, H M, which meaſures the angle B, w
be equal to the arch D E.

Farther, the angles of the triangle F D E, w
be meaſured by the oppoſite ſides of the trian
A B C; only that in the place of the great
angle E, its ſupplement muſt be taken.

Demonſt. The arch H B, and the arch A
are equal, being both quadrants, (by *Sup.*
therefore G H, which meaſures the angle D,
equal to the ſide A B, &c.

CORO

COROLLARY.

If in the triangle A B C, the three angles are known, I ſuppoſe, in oppoſition to it, another triangle D E F, wherein making *Fig.* 38. D F the ſupplement of the angle A, I give to F E the value of the angle C, and to D E the value of the angle B. Then ſearching, by the foregoing, the angles D, E, F, the angle E gives me the ſupplement of the ſide B C, the angle D, the value of the ſide A B, and the angle F, the value of the ſide A C.

Obſerve, throughout all Trigonometry,

That as thoſe analogies which have the ſine total for their firſt term, are moſt commodious for working, it will be convenient to change ſuch analogies as have it not, into others which have, which may be done by means of the following Reflections.

1. The ſine total A B, is a mean proportional between the ſine complement A C, and the ſecant A D. If therefore the following analogy were propoſed, *Fig.* 39.

The ſine complement of the latitude : ſine total : : ſine of the declination : ſine of the amplitude.

I ſhould change it into this,

Sine total : ſecant of the latitude : : ſine of the declination : ſine of the amplitude.

2. The ſine total A B, is a mean proportional between the tangent B D, and the tangent complement B C, ſince the triangles A B C, A B D, are ſimilar. If then the following analogy were propoſed,

Fig. 40.

> *Tangent of the greateſt declination of the ſun : ſine total : : the tangent of the preſent declination of the ſun : ſine of its diſtance from its neareſt equinoctial point.*

I ſhould change it to this,

> *Sine total : tangent complement of the greateſt declination : :* &c.

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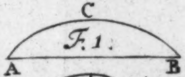
Trigonom: Part .1.

Plate.1.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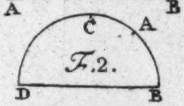
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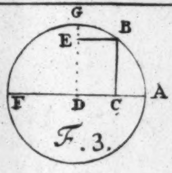
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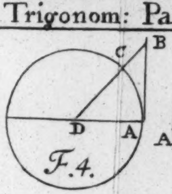
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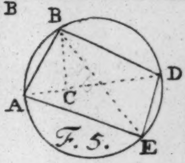
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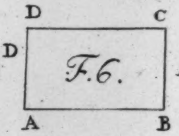
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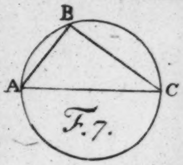
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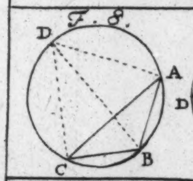
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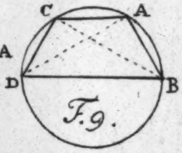
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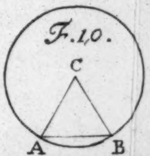
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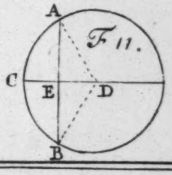
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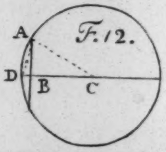
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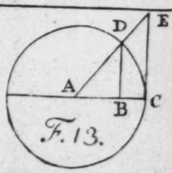
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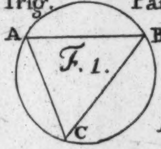
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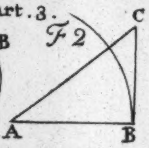
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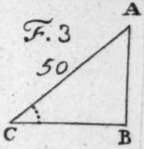
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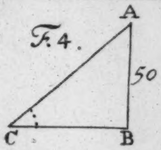
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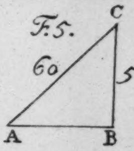
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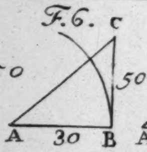
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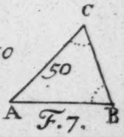
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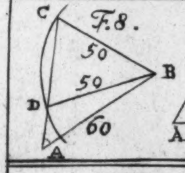
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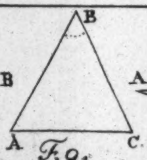
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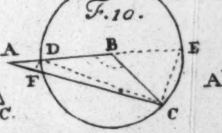
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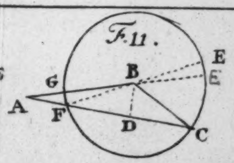
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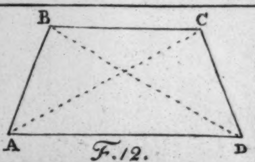
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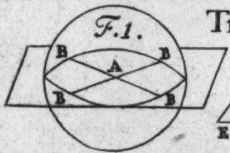
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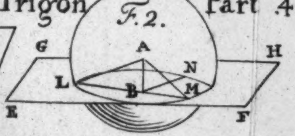
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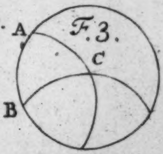
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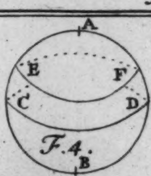
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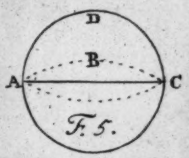
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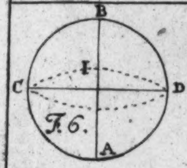
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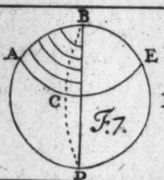
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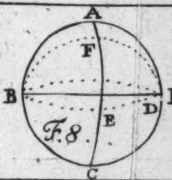
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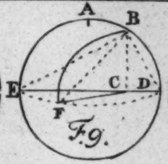
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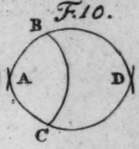
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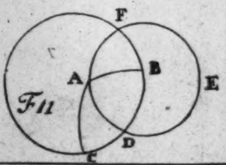
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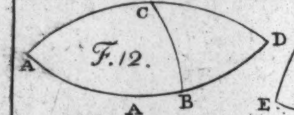
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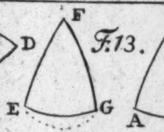
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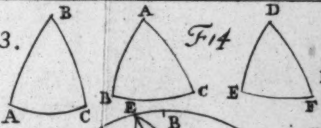
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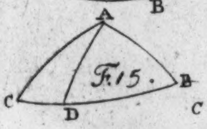
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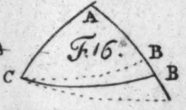
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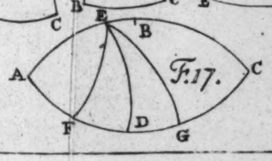
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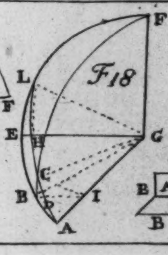
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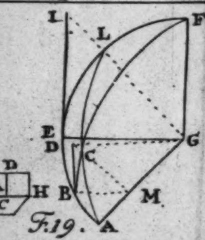
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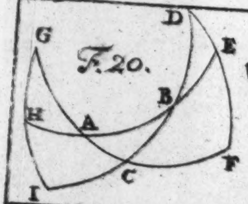
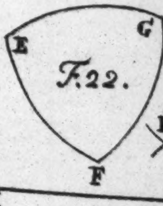
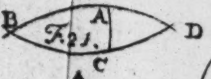
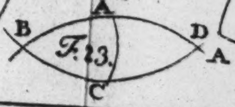
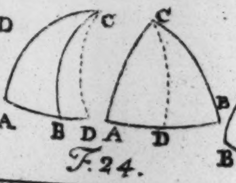
Trigonom: Part.4. Plate.2

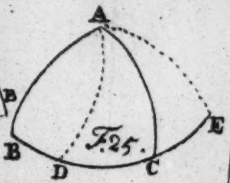


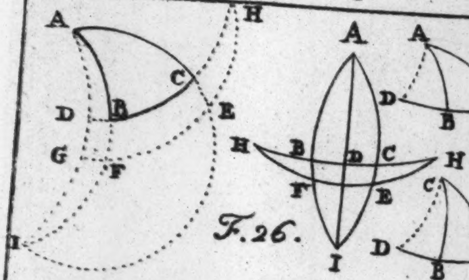


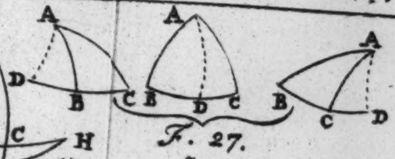


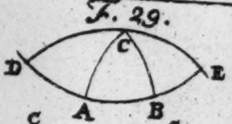


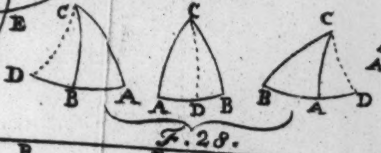


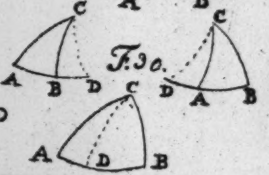


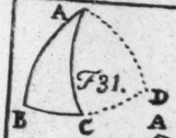


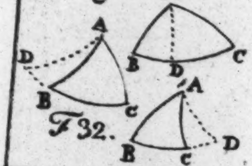


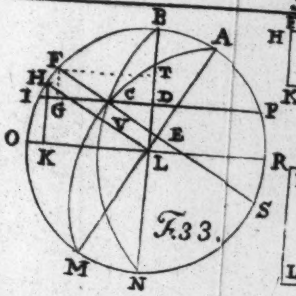


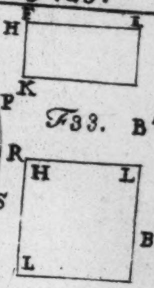


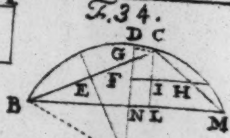


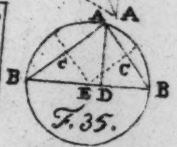


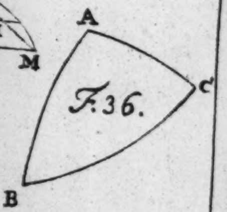


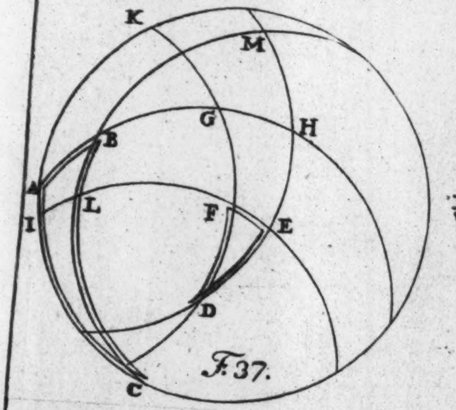


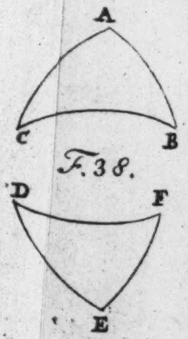


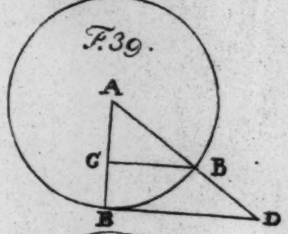


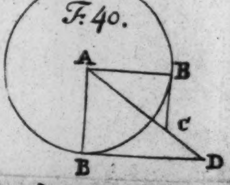












PRACTICAL GEOMETRY.

BY PRACTICAL GEOMETRY, is here meant in general, the art of meaſuring Lines, Surfaces, and Solids.

PART I.

Of LINES.

RIGHT Lines, or Lengths, are meaſured by compariſon with other right Lines, whoſe lengths are known or determined, as a *Fathom*, a *Foot*, an *Inch*, &c.

PROP. I.

To meaſure Heights.

If from the point A, a plummet B be ſuſpended by the line A B, and thro' the point D a line D C be drawn perpendicular to the line A B, the line C A will be what we call the height of the point A above the point D. *Fig.* 1.

This idea of height, though not quite exact, as ſhall be ſhewn in another place is ſufficient at preſent for us to conclude the height of any thing to be a line ſenſibly parallel to the thread of a plummet, in the place of obſervation.

2. Let it be propoſed to take the height of the tower A D. Place the foot of your *Fig.* 2. inſtrument, (which may be either a quadrant, or a ſemicircle divided into degrees, *&c.*) in any point of the horizontal line C D, whoſe length you may meaſure with your rule, then diſpoſing your inſtrument ſo that the plummet may play freely on the limb, look thro' the two ſights till you ſee the point A, the top of the tower; for then will the point A, and the two ſights be in the ſame right line A C, and the thread of the plummet F E being parallel to the perpendicular height of the tower A D, the angle E F C will be equal to the angle C A D, (by 15. 1.) which will be alternatively oppoſite to it, and conſequently the angle complement of the angle E F C will be equal to the angle C, the complement of the angle A. Meaſuring therefore the line C D with your rule, you may reſolve the triangle A D C, (by *Trig.* 12.) according to the following analogy.

As the ſine of the angle A : *to the line* C D :: *the ſine of the angle* C : *to the line* A D.

In the ſame manner you may find the length of the line C A.

You may alſo reſolve or meaſure the triangle A C D mechanically, thus : Lay down from any Scale of equal parts, ſo many parts on the line

E F, as you find feet or fathoms, *&c.* in the line CD, and drawing the perpendicular F G, lay down the line G E, ſo as to make the angle F E G equal to the angle C; for then the triangle EF G being ſimilar to the triangle A C D, you may conclude, that there are as many feet or fathoms, *&c.* in the line A D, as you find parts of the line F E, in the line F G; ſo that the line F E will ſerve as a ſcale, to meaſure all the lines of the triangle A D C.

3. If you were to meaſure the height of the rock A C, above the ſurface of the earth B, and could not apply your rule *Fig.* 3. to the horizontal line B C. Take two points, as B, D, any where in the line B C, ſo that you may meaſure the diſtance B D with your rule, and can ſee the point A thro' the ſights of your inſtrument; then at the point B take, as above, the angles B A C, A B C, and at the point D the angles D A C, C D A, which will furniſh you with the following analogies.

As the ſine of the angle B A D : *to the line* B D : : *the ſine of the angle* D B A : *to the line* A D.

Then, *As the ſine of the angle* C : *to the line* AD : : *the ſine of the angle* C D A : *line* A C.

Or mechanically; laying down upon the line EF ſo many equal parts as there are feet, *&c.* between B and D, and making the angle G E F equal to the angle B, as the angle H F G muſt be equal to the angle D; then from the point G, where the lines E G, F G meet, draw the perpendicular G H, and the figure H F E G, will be ſimilar to the figure C D B A, and the line F E will ſerve as a ſcale for the meaſurement, that is, all the lines of the figure H F E G may be meaſured by the ſcale from which the line F E was laid down.

4. If you cannot find two points in the ſame level, chuſe two points B, D, whoſe *Fig.* 4. diſtance you can meaſure at leaſt with a chord, and from both which you may ſee the point A, thro' the ſights of your inſtrument; then from the point B, take the angles D B C, A B C, B A C, and from the point D, the angles D A C, A D B, and you will have no trouble to reſolve the figure A D B C, or to make a ſimilar one E I G H, of which E I will be the ſcale.

5. In the ſame manner may be mea- *Fig.* 5. ſured the lines A D, A B, from the top of a tower, provided we know the line A C, or D B, or D C.

Prop. II.

To meaſure inacceſſible Diſtances.

Suppoſe you would know the diſtance betwixt two trees A, B, ſtanding on the other *Fig.* 6. ſide of a river: Chuſe two ſtations, as C, D, the diſtance between which you may meaſure; then planting your inſtrument at D, and taking care it lies truely horizontal, look thro' the ſights till you ſee the points C, A, B, ſo ſhall you find the angles A D B, B D C; do the ſame at the point C, to find the angles B C A, A C D, then will you be furniſh'd with ſufficient

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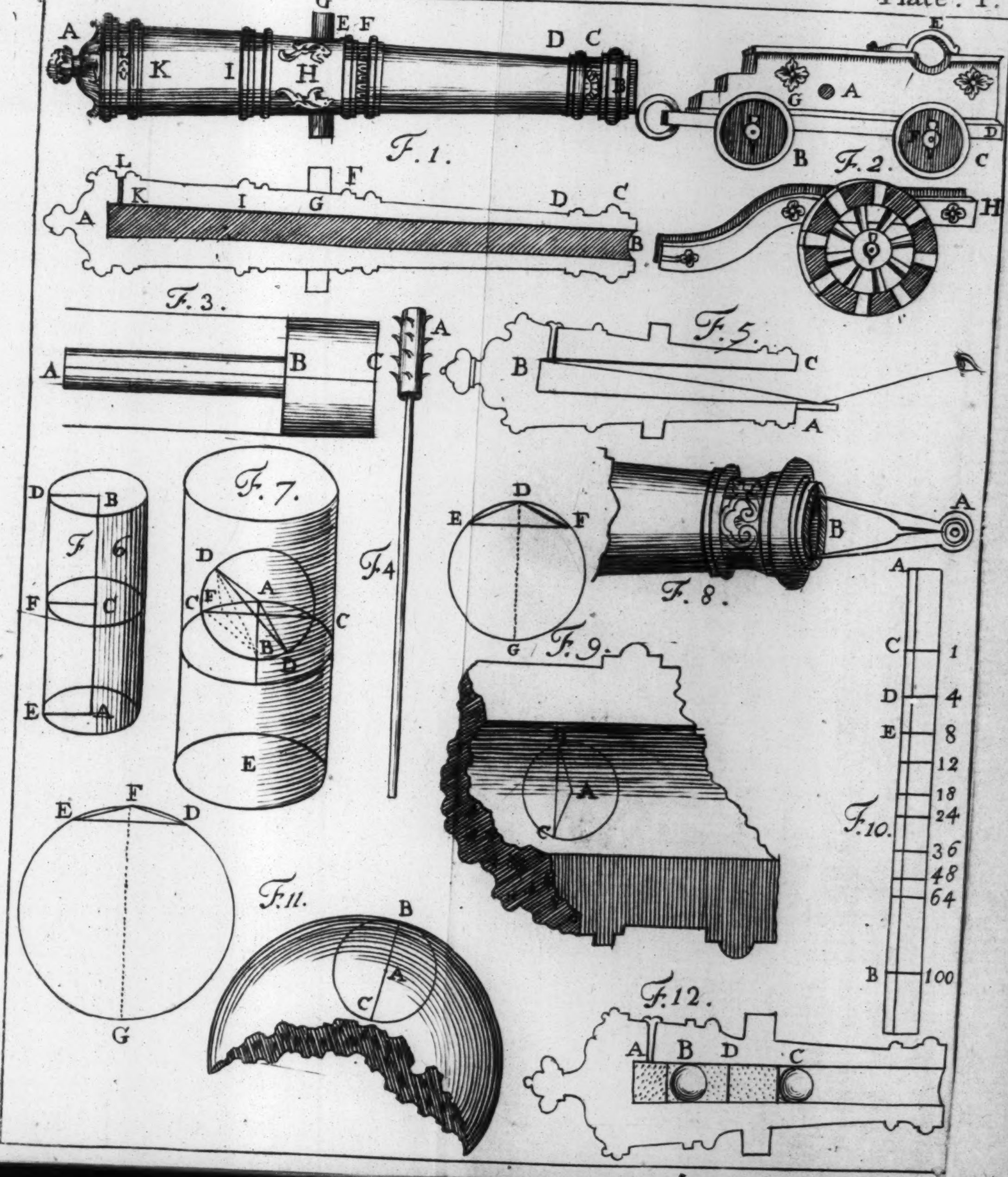
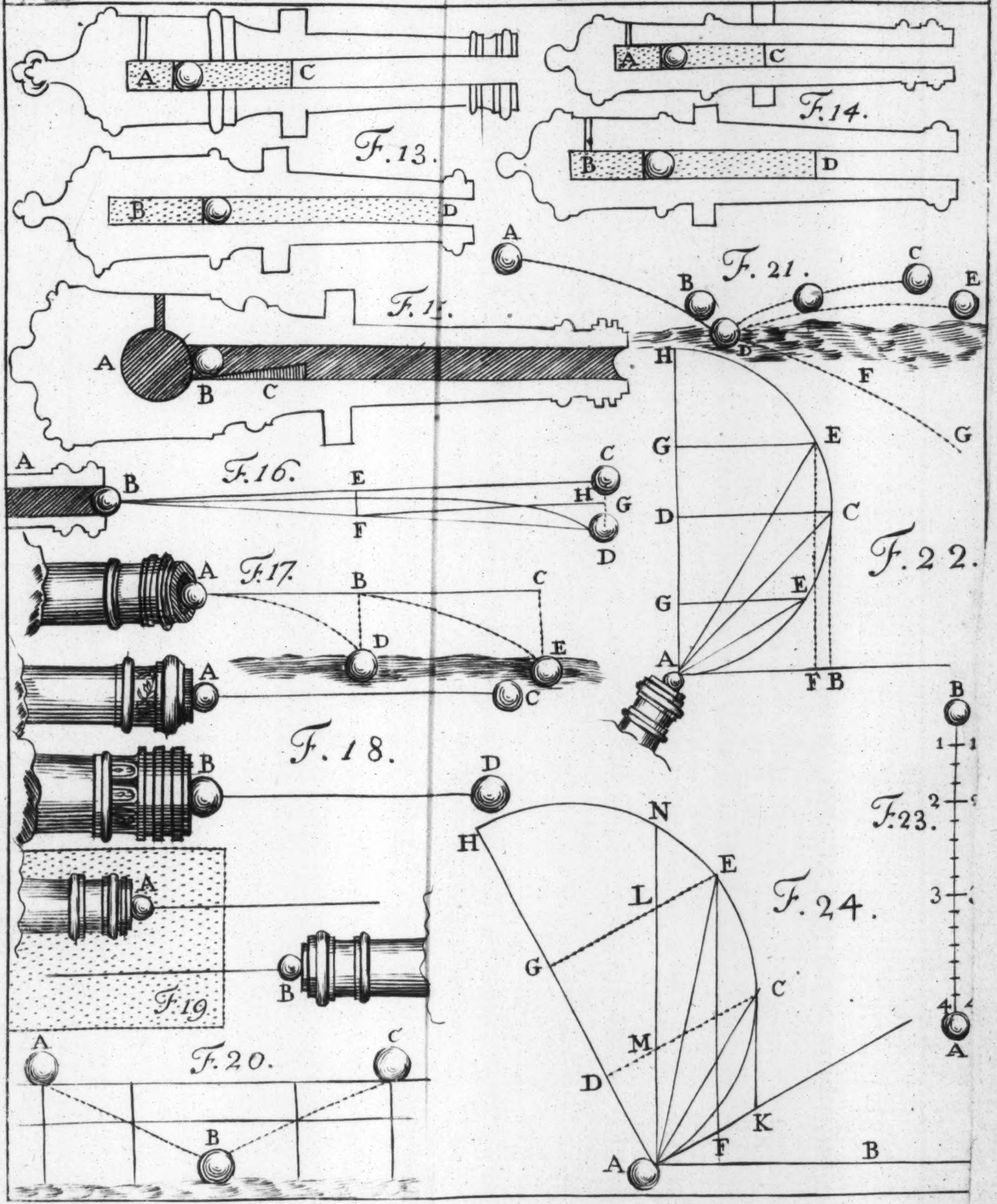
Gunnery

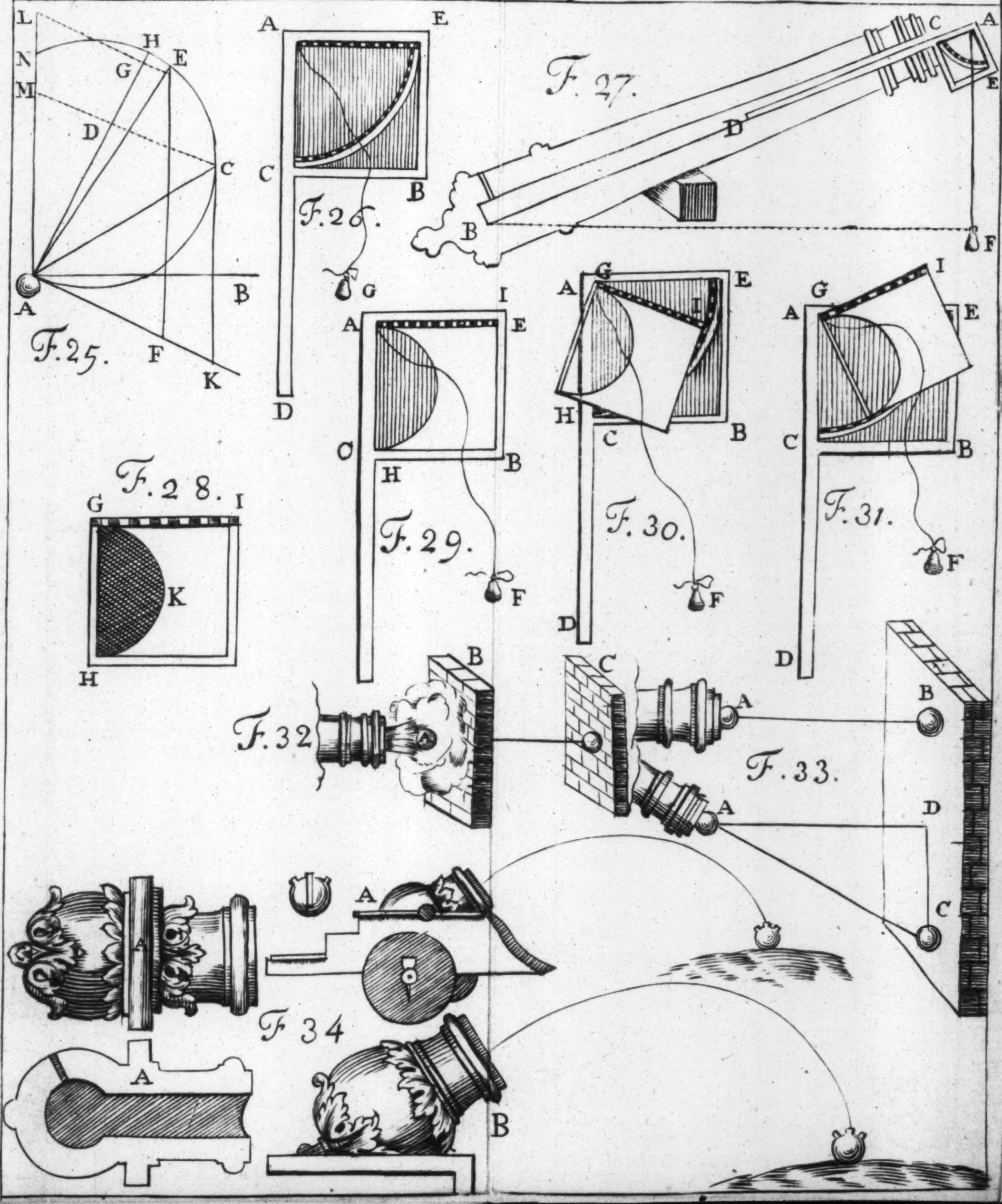
Plate. 1.



Gunnery Plate. 2.



Gunnery. Plate. 3.



data to reſolve the figure A B C D, or to make a ſimilar one G H F E, meaſurable by the ſame ſcale from which you laid down the line F E.

Prop. III.

To lay down the Ichnographical Plan of a place upon the ſpot.

An Ichnographical Plan is a figure ſimilar to that which the thing itſelf makes upon the earth's ſurface. Thus the Plan of a Houſe is a juſt repreſentation of the figure ſuch houſe makes upon the level whereon it ſtands.

1. Provide a ſcale divided into what parts you pleaſe, as ſuppoſe fathoms, feet, and inches, as in Fig 7, making your diviſions for fathoms greater or leſs in proportion, as you would have your Plan large or ſmall. *Fig.* 7.

The uſe of this ſcale is very eaſy: For if I would, for example, take upon it 3 fathoms 4 feet 2 inches, I place one foot of my compaſſes on the point A, common to the line of 3 fathoms, and that of 2 inches, and extend the other foot to the point B, where the line of 2 inches cuts that of 4 feet, and the diſtance A B, will evidently be 3 fathoms 4 feet 2 inches. *(See in the Appendix the deſcription and uſe of Diagonal Scales.)*

2. Several inſtruments are uſed for taking the angle made by two walls, but the beſt ſort is that called a *Metragon*, made of two rulers, join'd ſo at C, as that they may open and ſhut; round the point C is de- *Fig.* 8.

ſcribed a circle divided into degrees, and on the point C, or center, is fixed a ſmall index, which, as the rulers open or ſhut, runs thro' the diviſions, and ſhews exactly the diſtance they ſtand at.

The uſe of this inſtrument is, by applying one ruler to one wall, and the other to the other, either on the inſide, or the out, and the index will ſhew the angle they make with each other.

3. Suppoſe it were required to take the ichnographical plan of the building A B C. *Fig.* 9. Meaſure the length D E of the wall A, and taking the ſame meaſure from your ſcale, ſet it upon the line H I; then taking the length of the wall B, and the angle it makes with the wall A, ſet off from your ſcale the length of the wall B, upon I K, laying down the line I K, ſo that the angle H I K, may be the ſame that the wall A makes with the wall B; do the ſame with all the other walls, and you will produce the figure H I K L M, which will be the plan of the building A B C. The reaſon of which is evident.

4. But becauſe the multitude of walls and angles to be taken in a large building, *Fig.* 10. may cauſe ſome error to ſlip in, and ſo hinder the cloſing of the figure, obſerve the following method: Suppoſe a plan were to be taken of the building R. Extend the chord A B, divided into fathoms, feet, and inches, along one ſide, as A E, the length of which you may tranſport from your ſcale upon H I, which you may continue or produce as an obſcure line, to what length you pleaſe. Farther, having a large ſquare, ſlide one of its ſides along the chord, till

the other exactly anſwers to the point D, then take from your ſcale the diſtances A E, and E C, and having laid them down upon H I M, ſet likewiſe the diſtance C D upon the perpendicular M N, and the point N in the plan will repreſent the point D. Do the ſame for all the other points, as you may ſee in the figure, ſo ſhall the figure V be the exact plan of the building R.

In this manner may the plan of a City, with its ſeveral ſtreets, be eaſily taken.

Prop. IV.

To take Ichnographical Plans at a diſtance.

Suppoſe a plan were to be taken of the place A. Chuſe two ſtations, as B, C, as far from each other as you can conveniently, *Fig.* 11. (meaſuring the diſtance) from whence you may ſee the moſt angles of the place, as E, F, G, H, I, K, then fixing your inſtrument horizontally at the point B, take (by *Prop.* 2.) the angles E B C, F BC, G B C, H B C, I B C, K B C, do the ſame at the point C, for the angles K C B, I C B, H C B, *&c.* then taking upon your paper two points M, N, to repreſent the points B, C, make the angles O M N, P M N, R M N, S M N, *&c.* equal to the angles E B C, F B C, G B, C, *&c.* and the angles O N M, P N M, R N M, equal to the angles E C B, F C B, G C B, *&c.* and the lines M O, N O, meeting at the point O, as the lines M P, N P. at the Point P, *&c.* will form the figure O P R S X T. ſimilar to the figure E F G H I K.

To proceed with the plan, you muſt take two other ſtations, D, L, which are repreſented in the plan

plan by two other points, as Z, Q, in making the angles T N Z, Z T N; equal to the angles K C D, D K C, and the angles Z T Q, Q Z T, equal to the angles L K D, L D K; it will be eaſy thus to take as many ſtations as you pleaſe, doing in all reſpects as at the ſtations B, C, to take in the whole circumference of the place. Each ſide may then be meaſured by the ſcale, from which you took the lengths of your ſtationary lines.

Note, 1. It will be ſtill more eaſy to take plans of fields, and of roads, where you only fix your inſtrument at ſome principal points, which will ſerve as ſtations for taking thoſe between them.

2. When we are obliged to take the plan of any place upon the ſea, we uſe the Compaſs for taking the angles.

PART II.

To meaſure SURFACES.

SURFACES, or Superficies, are meaſured by compariſon with certain ſquares, denominated from their reſpective ſides, as ſquare fathoms, or ſquare feet, *&c.*

PROP. V.

To meaſure plain Surfaces.

1. To find the area or content of the rectangular paralellogram A B C D, multiply its baſe A B by its height A D, and the product will give you the content, that is, the ſquares it contains: and as every other parallelogram, as A B F E, is equal to a rectangle of the ſame baſe and height A B C D, (by 35. 1.) its content will be found by multiplying its baſe by its height, as before, tho' it be not a rectangle. *Fig.* 12.

2. The triangle A B C, being the half of a parallelogram of the ſame baſe and height, (by 37. 1.) to have its content you muſt either multiply its baſe by half its height, *Fig.* 13.

height, or its height by half its baſe, or multiplying the whole baſe by the whole height, take half the product.

3. If in the trapezium A B C D, the lines A B, C D, are parallel, multiply the half of A B, and the half of C D, by the height E F. But if they are not parallel, multiply the diagonal A C by half the ſum of the perpendiculars D E, B F.

Fig. 14.

Fig. 15.

REMARK.

Thus Serveyors meaſure land, and thus Artificers meaſure their works, as pavements, roofings, and floors, *&c.* Theſe Propoſitions may alſo be applied to the Military Art, in finding what number of men compoſe a battalion or ſquadron; for it is but multiplying the number of men in rank by thoſe in file, and you have the anſwer.

4. To find the area of a regular polygon, as A; draw from the center A, the perpendicular A B, to one of its ſides H C, which being multiplied by half the circumference, or ſum of the ſides, will give the content: For if I R be taken for the height of a triangle equal to B A, and the baſe I P be made equal to the circumference C D E F G H, the triangle I R P will be equal to the polygon A.

Fig. 16.

5. As a circle may be taken for a polygon of an infinite number of ſides, to which its radius is perpendicular, its area may be alſo found by multiplying its radius by half its circumference. And thus we ſhould find the quadrature of a

circle, (which could never yet be obtain'd) if we could exactly determine the proportion between the circumference and the diameter. We can only find, by calculation, that the circumference is to the diameter nearly as 22 to 7, or 3,14159 to 1.

To meaſure a ſurface whoſe boundaries are ſome of them ſtrait lines, and others curve, divide it into ſeveral triangles, *Fig.* 17. which, by reaſon of their ſmallneſs, may paſs for rectilinear, without any conſiderable error.

Prop. VI.

To meaſure curve Surfaces.

1. To find the ſurface of a rectangular cylinder, multiply the circumference of its baſe, by its height; *Fig.* 18. for if you imagine the curve ſurface of a cylinder extended to a plain, it would become a rectangular parallelogram, whoſe height would be the height of the cylinder, and its baſe the circumference of the baſe of the cylinder.

If the cylinder A is not rectangular, you muſt multiply the circumference of the baſe by the line B D; *Fig.* 19. becauſe its ſurface will be an infinite number of parallelograms, having all the ſame height B D, and whoſe baſes all together will make up the circumference of the baſe of the cylinder.

2. If the circumference of the baſe of a right cone A, be multiplied by *Fig.* 20. half its ſide E F, it will produce the area of its ſurface; which is nothing elſe but an

infi-

infinite number of triangles, whoſe height is E F, and all whoſe baſes together form the circumference of the baſe of the cone.

L E M M A I.

If a right cone be cut by a plane parallel to its baſe, the common ſection D will be a *Fig.* 21. circle, whoſe radii will all have the ſame proportion to M A, the radius of the baſe, as C D to C A, or as D O to M A, (by 4. 6.) and as this ſaid ſection will be perpendicular to the axis C D A, (by 1. 11.) the part D C of the divided cone, will itſelf be a right cone.

COROLLARY.

If H G be ſuppoſed equal to the circumference of the circle A, and L F equal to the ſide C M; if alſo E F be equal to the ſide C O, then will I E K, parallel to H G, be equal to the circumference of the circle D. For the circumference of the circle D, is to that of the circle A, as D O to M A, or C D to C A, or as I K to G H, which was ſuppoſed equal to the circumference of the circle A. Thus the triangle F I K, will be equal to the ſurface of the cone D C, and the trapezium I K G H, will be equal to the ſurface of the ſection D A.

In the ſame manner, if we ſuppoſe the baſe V, equally diſtant from the baſe D, and the baſe A, its circumference will be found equal to the line R P of the trapezium, equally diſtant from the line H G, and the line K I. But the trapezium G H I K is equal to a rectangular parallelogram of the ſame height, having R P for its baſe; therefore the ſurface of the ſection of a cone, is equal to a rectangular parallelogram, whoſe baſe is equal to the circumference of its mean baſe V, and height equal to the ſide O M.

LEMMA II.

Suppoſe the ſemicircle A K G, touch'd by the line O B in the point C, and by the line A E in the point A; further, ſuppoſe C B, C O, equal. Draw B E, O N, and C M, parallel to A G; and L K, X F, parallel to A E; if then the whole figure be ſuppoſed to be turn'd about the perpendicular L K, whoſe point L is the center of the ſemicircle, the line O E will deſcribe the ſurface of a right cylinder, whoſe radius will be O N, or L A, and its height X F; and the line O B will deſcribe the ſurface of a ſection of a cone, the radius of whoſe mean baſe ſhall be C M, and its ſide O B; I ſay, that theſe two ſurfaces will be equal. Draw L C.

Fig. 22.

Demonſtrat. The rectangular triangles E O B, C M L, are equiangular, for the angle B C M, which is equal to the angle B, (by 15. 1.) the complement of the angle O, is alſo the complement of the angle L C M. Then (by 4. 6.) E O, or X F : O B : : C M : C L, or A L; and ſince the radii are in the ſame ratio as the circumferences, (by 1. 12.) X F, or O E : O B : : the circumference of the mean baſe C M : the circumference of the baſe A L. Therefore (by 15. 6.) the rectangle under O E, and the circumference of the baſe A L, which is the ſurface of the cylinder,

der, is equal to the rectangle under O B, and the circumference of the mean baſe C M, which is the ſurface of the ſection of the cone.

If the point B falls in the line L K, the ſame thing will follow, tho' the cone be not *Fig.* 23. cut. For the triangles X B N, C L M, will be ſimilar, and conſequently B N, or O E : X B : : C M : C L.

COROL. I.

If about the ſemicircle A be circumſcrib'd the polygon D B E, and the perpendiculars *Fig.* 24. A B; D C, E F, be drawn upon the diameter D E; if, further, the whole figure be ſuppoſed to be turn'd about the line AB, the ſides of the polygon will deſcribe ſurfaces of cones, which all together will be equal to the ſurface of the cylinder deſcrib'd by the line D C. But the ſemicircle is a polygon of an infinite number of ſides; therefore the ſurface of the demiglobe, which it deſcribes, will be equal to the ſurface of the cylinder deſcrib'd by th line D C.

COROL. II.

If the ſurface of the cylinder, and that of th demiglobe, be each of them divide *Fig.* 25. into ſeveral parts by parallel plane each part of the one, will be equal t the correſpondent part of the other.

PRO

Prop. VII.

To find the ſurface of a demiglobe, multiply its radius by the circumference of one of its great circles, or by $\frac{22}{7}$ of its diameter. *Fig.* 26. In the ſame manner, to meaſure the concavity of a vaulted or ſpherical roof, paſs a circle thro' its ſummit D, and thro' two points E, C, diametrically oppoſite; then multiply the height A D by $\frac{22}{7}$ of the diameter of ſuch circle. All which follows from the preceding *Corol.*

We take no notice here of ſurfaces deſcribed by the circumvolution of parabolas, hyperbolas, ellipſes, &c. nor of bodies bounded by thoſe figures, as not being neceſſary to our preſent deſign.

⁂

PART III.

To meaſure SOLIDS.

SOLIDS are meaſured by compariſon with certain known cubes, denominated from their reſpective ſides; as cubical fathoms, cubical feet, *&c.*

PROP. VIII.

Fig. 27. 1. To find the ſolidity or content of the rectangular parallelipepidon A; multiply the area of its baſe B C D E, by its height B F, becauſe if its baſe contains 4 feet ſquare, and its height 2 feet, it will contain two layers of 4 feet ſquare each.

Fig. 28. 2. If the parallelipepidon B is not rectangular, you muſt ſtill multiply its baſe by its height G H, becauſe it is equal to a rectangular one of the ſame baſe and height.

Fig. 29. 3. To find the content of a priſm, as A, multiply the area of its baſe by its height, it being the half of the parallelipepidon A B E C, which is to be underſtood of all kinds of priſms, as well as triangular ones, ſince they all may be divided into triangular priſms; and further, ſince cylinders are priſms of an infinite number of ſides, we alſo multiply the area of their baſes by their height, to have their content.

4. The area of the baſe of a pyramid A, being multiplied by the $\frac{1}{3}$ of its perpendicular height A E, will give its content, *Fig*. 30. becauſe it is but the one third of a priſm of the ſame baſe and height (by 7. 12.) The ſame is to be underſtood of cones, which are pyramids of an infinite number of ſides.

5. A globe may be conſidered as an infinite number of pyramids, whoſe vertexes meet in the center, and whoſe baſes form the circumference: the radius of the globe will be the common height of the pyramids; conſequently therefore, multiplying the ſurface of the globe by $\frac{1}{3}$ of its radius, will produce its content, that is, the content of all the pyramids compoſing it.

6. In the ſame manner the ſector of a globe is equal to a cone, having the radius of a ſector for its height, and its baſe equal to the convex ſurface of the ſector. Thus may a ſector, or any other part of a globe be eaſily meaſured.

7. To meaſure the ſection of a cone, as A B, prolong its ſides C D, E F, till they meet in the point K; then meaſure the *Fig*. 31. cones A K, B K, and ſubtract the little one from the great one; the remainder will be the content of the ſection A B.

To avoid a multitude of operations, the common practice is, to multiply the baſe A, and the baſe B, by the half of their diſtance A, B, tho' this way is not altogether exact.

In order alſo to find more readily the content of the baſes by their diameters, we make *Fig.* 32. uſe of a rule A B, divided in the following manner. Take A D equal to 1 foot 3 inches, and a little more than three lines, (which is the meaſure of the diameter of a circle equal to a foot ſquare) and ſet A C equal to A D, perpendicular upon A B: it is then plain (by 47. 1.) that C D will be the diameter of a baſe of 2 feet; make then A E equal to C D, and A F equal to C E, and A G equal to C F, *&c.* and ſo will you obtain the ſeveral diviſions of your rule; and in the ſame manner alſo may you lay down the halfs, and quarters. Thus we meaſure Veſſels, Maſts, Yards, Piles, *&c.*

8. Other irregular bodies are meaſured by dividing them into priſms, and pyramids, and oftentimes the ſhorteſt ways are preferr'd to the exacteſt. Thus ſome gauge a ſhip as if it were a priſm, by multiplying the length of its keel into a triangle, whoſe height is the depth of the ſhip, and its baſe the mean width, or length of the midſhip beam.

PART IV.

Some PROBLEMS.

THIS Part in the Original contains five Problems, the firſt and ſecond of which being the conſtruction of the *French* ſector, and the proportional compaſſes, and the fourth the manner of inſcribing regular polygons in a circle; as I have more largely treated of the firſt in the Appendix to this work, and included the laſt in the extract I have added of the fourth Book of *Euclid*, omitted in the *French*; I ſhall therefore here only add the two following Problems.

PROBLEM I.

To make plain figures equal to others given.

1. To make a parallelogram equal to a triangle A B C, let its baſe E B be equal to half the baſe of the triangle, and make *Fig.* 33. it between the parallels E B, C D, (by 41. 1.) But if the parallelogram muſt have a determin'd line L for its baſe, continue C D, till D K be equal to L, then drawing the K B, till it meet C E in the point F, ahd finiſhing the paral-

lelogram C K F H, its complement B G H I will be the parallelogram required.

Fig. 34. 2. If you divide the right-lin'd figure given A B C D E, into three triangles, and change thoſe triangles into three rectangles having the ſame baſe F G, (by the preceding) placing them one upon another you will make a rectangle equal to the rectilinear figure given.

3. If you ſeek a mean proportional between the baſe F G of the rectangle F G I H, and its height G H, (by 13. 6.) the ſquare of ſuch mean proportional will be equal to the rectangle F G H I, and conſequently to the rectilineal figure A B C D E.

PROBLEM II.

Of the diviſion of right-lin'd plain figures.

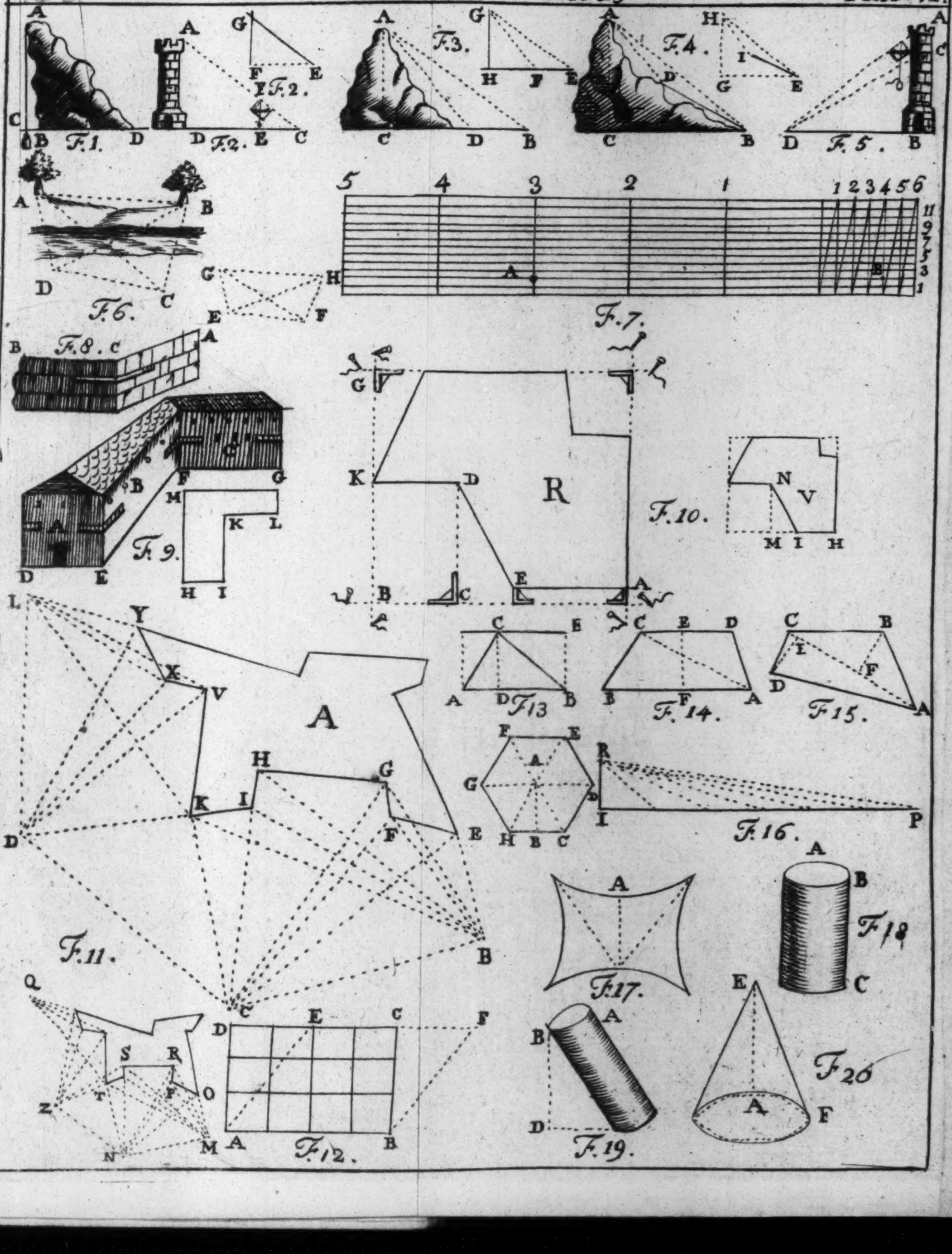
Fig. 35. 1. To divide a triangle into any number of parts, either equal or unequal. in any proportion given; divide its baſe A B into ſuch number of parts, and in ſuch proportion; then draw lines from the point C, to each diviſion: ſo ſhall the triangle be divided in the proportion required, (by 1. 3.)

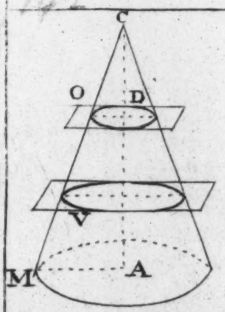
Fig. 36. *Fig.* 37. 2. In the ſame manner to divide the parallelogram A B C D, divide its parallel ſides A B, CD. The ſame is to be done with a trapezium, as A B C D, whoſe ſides A, C D, are parallel.

3. If the rectilinear figure A B C D E be to be divided in any given ratio; divide it into triangles, and make three rectangles of the ſame height, equal to the three triangles, *viz.* LN, NO, OP, (by *Prob.* 1.) then divide the line L P according to the given ratio in the point R, and ſuppoſing that point falls in the baſe L N of the parallelogram, equal to the triangle CDE, divide D E in the point S, in the ſame ratio as L N in the point R, and the line C S will divide the rectilinear figure in the ratio given, as the perpendicular R Q divides the rectangle M P in the ſaid ratio, (by the preceding) ſince the triangle C S E will be equal to the rectangle L M Q R, both the one and the other having the ſame ratio.

Fig. 38.

Practical Geometry





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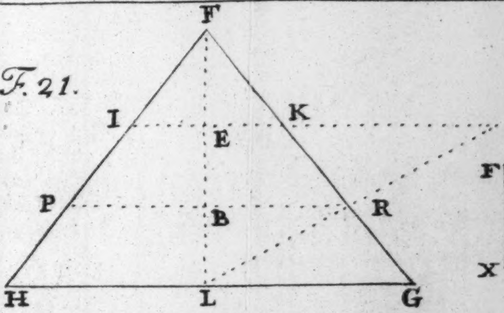

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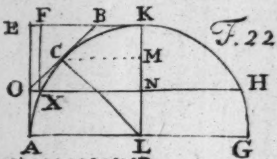

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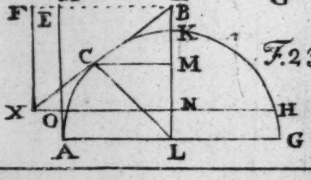

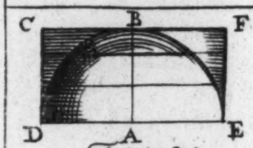

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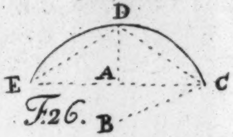

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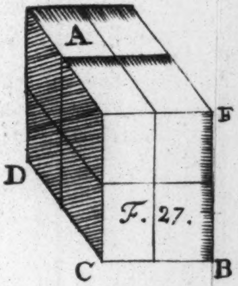

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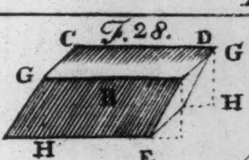

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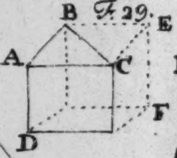

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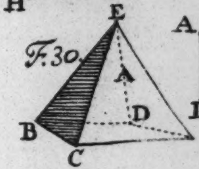

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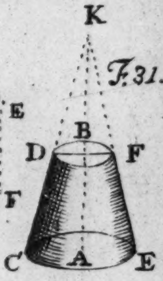

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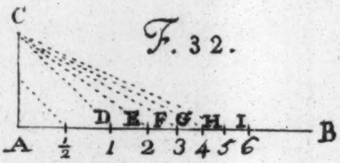

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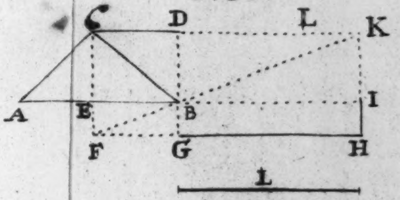

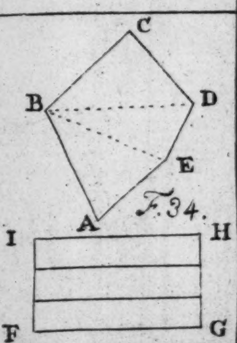

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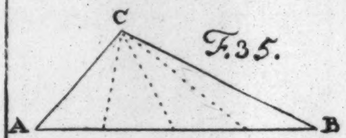

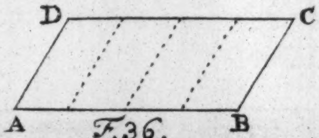

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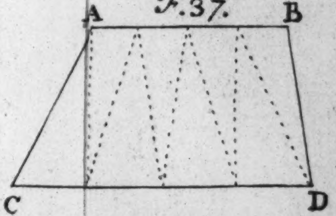

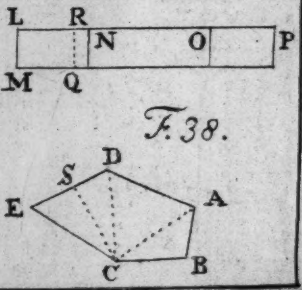

F. 38.

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MECHANICKS.

DEFINITIONS.

1. BY *Mechanicks*, is here meant the art of moving or raiſing heavy bodies with facility.

2. We call *Weight* or *Gravity* that inclination of heavy bodies to deſcend, or approach the center of the earth.

3. By *Motion* we underſtand the ſucceſſive progreſſion of a body from one place to another, and of the quantity of ſuch motion we judge, 1. By the magnitude of the body moved. 2. By the length of the ſpace it runs through in a limited time.

4. *Velocity* therefore is nothing elſe but the ſpace the body runs through in a time determined.

5. *Power*, is that which gives the motion, whether it be an animated thing, as the hand of man, or inanimate, as the weights of a clock.

PHYSICAL SUPPOSITIONS.

1. The Reſiſtance found in moving a body proceeds chiefly from two cauſes: 1. The body we would move has ſometimes beforehand a contrary motion to that we would give it, as when we would raiſe a weight which has a natural tendency to deſcend. 2. At other times, the body we would move cannot ſtir without the partition or diviſion of ſomething containing it, as when we would move a body in the water.

2. If two bodies are equal in all reſpects, an equal power is neceſſary to give them an equal velocity of motion, (by *Def.* 3.) But to make one body move with twice the velocity of another, a double power muſt be applied, (by *Def.* 3.) therefore every thing elſe being equal, the power muſt be increaſed in the ſame ratio or proportion with the velocity.

3. A power can only communicate ſo much motion as is in itſelf.

4. Weight or Gravity being nothing elſe but a inclination or tendance towards the center of th earth; that motion increaſes always in proportio with the weight.

5. A power cannot move a body, unleſs i force be greater than the reſiſtance, or contra motion of the body; and as the reſiſtance of body increaſes in proportion to the velocity of i motion, (by *Def.* 3.) it is evident the force of t power ought to increaſe in the ſame proporti

with the velocity of the body, all other circumſtances remaining equal. The following Axiom may therefore be looked upon as infallible.

A Power is capable of moving a Body, when the force of that power has a greater proportion to the contrary force of the body, than the velocity of the body to be moved has to the velocity of the power, and not otherwiſe.

Upon this principle then, (which exactly agrees with *Definition* 3.) the whole doctrine of Mechachanicks may be ſaid to depend, without ſearching after any others, as many authors do to no purpoſe.

PROP. I.

If the weights A and B are equal in all reſpects, and are hung on the two ends of a balance horizontally ſuſpended at the point C, ſo that the parts C A, C B, are exactly equal, the weights will remain in equilibrium. *Fig.* 1.

Demonſt. The weight A cannot deſcend to the point D, without raiſing the weight B to the point G, which is impoſſible, (by *Ax.* preced.) ſince its force has no greater proportion to the contrary force of B, than the ſpace of velocity F G of B, has to the ſpace or velocity, E D of A.

REMARK.

It is true, that as the deſcent or aſcent of heavy bodies are meaſured by lines tending towards the center of the earth, as C B, F A, the weight B will deſcend more than the weight A will riſe. For H G : G F :: *Fig.* 2.

A D: F A, and H G: G C : : E B, or A D: B C; then ſince G C is greater than G F, B C will be greater than F A. But as this difference will ceaſe upon placing the beam duly horizontal; ſo if the ſide on which the weight A hangs be inclin'd, then will the weight A have the advantage of the weight B, for the ſame reaſon.

2. If the weight A is double the weight B, and if reciprocally the diſtance C B is double the diſtance C A, the weights will be ſtill in equilibrium. *Fig.* 3.

Demonſt. The force of the weight A will be to the force of the weight B reciprocally, as the velocity or ſpace run thro' of the weight B, *viz.* D F, to the velocity or ſpace E G, run thro' by the weight A; therefore (by *Ax.*) the weight A cannot raiſe the weight B, nor the weight B the weight A.

In this caſe it is alſo neceſſary, that the beam be perfectly horizontal, for the ſame reaſon as before.

3. It is eaſy to conclude, that if the velocity of the weight B increaſes in greater proportion than the gravity of the weight A, the weight B will conſequently raiſe the weight A.

COROLLARY.

From what has been ſaid it may be concluded, that the whole art of Mechanicks conſiſts in diminiſhing the vilocity of the weight to be moved, at the ſame time that we increaſe the velocity of the power cauſing ſuch movement.

This may be done many ways, of which theſe that follow are the chief.

PROP. II.

Of the Lever.

1. If the bar A B be ſo reſted on the point C, that the part B C be ſhorter than the part A C, when the power A deſcends, the weight B will be raiſed; but the velocity of the power A will be greater than the velocity of the weight B, in the ſame proportion as C A is greater than C B; therefore the reſiſtance of the weight B will diminiſh in the ſame proportion with regard to the power A, (by *Cor.* preced.) that is to ſay, its reſiſtance will become to what it was before, as C B to C A. *Fig.* 4.

We call the bar A B in this caſe, a *Lever of the firſt kind*

2. If the bar A B be reſted upon the point A, and the weight be hung at C, its reſiſtance with regard to the power B, will diminiſh in the ſame proportion as C A is leſs than B A, becauſe its velocity will be diminiſhed in that proportion. For ſince the arch of the aſcent of the power B will have A B for its radius, the arch of the aſcent of the weight C, will have for its radius but C A. In this ſecond caſe, A B is called a *Lever of the ſecond kind.* *Fig.* 5.

It will be eaſy to refer to theſe two kinds of Levers, ſeveral ſorts of Inſtruments, as pinchers, ſciſſars, *&c.*

PROP. III.

Of inclined Planes.

If the weights A and D are faſtened to the chord A E D, and the weight D deſcends by the vertical line D E, whilſt the weight A is raiſed up the inclined plane A E; the velocity or ſpace run thro' by the weight A, when come to E, ſhall be meaſured by the line E D, ſince that is the true height it is raiſed; and the velocity or ſpace run thro' by the weight D, ſhall be meaſured by the line A E; and conſequently the reſiſtance of the weight A will diminiſh with regard to the power D, in proportion as D E to E A; that is to ſay, that upon the inclined plane A E, the reſiſtance of the weight A, will be to its own natural reſiſtance as E D to E A, ſuppoſing no impediment by friction. *Fig.* 6.

Wedges, and nails are to be referred to inclined planes.

PROP. IV.

Of Pullies.

1. The pullies A being fix'd, do not at all diminiſh the reſiſtance of the weight C, with reſpect to the power B, becauſe they make no alteration in their velocity. On the contrary, the friction of the chord paſſing thro' the pullies, increaſes the reſiſtance of the weight; therefore ſuch fix'd pullies are never uſed *Fig.* 7, 8.

uſed but for the more eaſy application of greater powers.

2. To leſſen the friction of the chords, wheels are placed in the pullies; which the bigger they are, the better, becauſe their circumferences have thereby the greater velocity with regard to the friction of the axis, beſides that they make fewer turns.

3. If the weight A be faſtened to the moveable block A, and a rope being fix'd to the immoveable block B, paſs ſeveral times thro' both, it is evident that the power C, being applied to the end of the rope, cannot deſcend without raiſing the block A; but to raiſe it the whole length of the line A B, the power muſt deſcend as many times the length of the ſame line, as there are returns of the rope from one block to the other. So that the velocity of the weight A, will be to the velocity of the power C, as A B to the ſeveral turns of the rope; and conſequently the reſiſtance of the weight will diminiſh in the ſame proportion, with regard to the power C. *Fig.* 9.

4. It is eaſy to perceive how by this means the reſiſtance of weights may be infinitely diminiſhed, *viz.* by multiplying the turns of the rope, or increaſing the number of pullies, as in *Fig.* 9. wherein the reſiſtance is 4 times leſs with regard to the power C, which has 4 times its velocity, but becomes 16 times leſs with regard to the power D, which has 16 times a greater velocity.

5. For the ſame reaſons, the weight A will have its whole force upon the rope which ſuſtains the immoveable block B, becauſe it has the ſame velocity with the block B; for if the block B be let looſe, it will fall thro' juſt the ſame ſpace as the weight itſelf would. But the weight A lays but $\frac{1}{4}$ of its ſtreſs upon the rope which faſtens the end B to the block, becauſe if the ſaid end be unlooſed, it will have 4 times more velocity than the weight; ſo alſo the rope faſtening the end H to the block F, will bear but the $\frac{1}{16}$ of the ſtreſs of the weight, becauſe that end will have 16 times the velocity of the weight.

PROP. VI.

Of Capſtons, Wheels, &c.

1. When the power C, at the end of the bar A C, has turn'd the capſton A B, the *Fig.* 10. rope D F will be alſo turn'd round the axle D, and will have ſo far advanced the weight E; but at every turn of the capſton the power deſcribes the circumference of a circle, whoſe radius is A C, and the weight E is only ſo far advanced as is one turn of the rope about the axle D; and conſequently the velocity of the power will be to the velocity of the weight, as the radius C A to the radius of the axle D, and the force of the power C will be augmented with regard to the weight E, in the ſame proportion.

2. It is eaſy to apply the ſame conſi- *Fig.* 11. deration to Wheels, to Cranes, *&c.* But *note*, the power of wheels may be infi-

nitely increaſed by mixing pinions, or ſmall wheels, among the great ones. For example, if the radius of the axle of the vertical wheel A, which has 72 teeth horizontally placed, is but a fourth part of the radius E F of the handle D, and if the wheel C has but 6 teeth, which take the teeth of the wheel A, when the power D ſhall have made 12 turns, the wheel A, and its axle, will have made but one, and the weight G will be rais'd but one turn of the chord about the axle H B; therefore the velocity of the power D will be to that of the weight G, as 12 turns of the handle to one turn of the axle H, or as 12 to $\frac{1}{4}$, or as 48 to 1; and conſequently the power D needs only be the $\frac{1}{48}$ part of the weight G.

3. A thouſand machines may be made by the multiplication of wheels; but great care muſt be taken that the friction be as little as poſſible.

Prop. VII.

Of the Screw.

If the powers D turn the ſcrews B, at each turn the weight A will be rais'd one notch, and the power will deſcribe the circumference of a circle whoſe, radius will be B D; and conſequently the reſiſtance of the weight A, will be diminiſh'd with regard to the power D, in proportion as the height of the notch is to the circumference of the circle, whoſe radius, as was ſaid, is B D.

Fig. 12.

PROP. VIII.

Of Liquors.

If the tube A is quadruple the tube B, with which it has a communication by the channel C, and any liquor be put into it, it will riſe to the ſame height in both tubes. *Fig.* 13.

Demonſtrat. If there be 4 pounds weight of liquor in the tube A, there will be 1 pound in the tube B, ſince both tubes will be equally fill'd. But the 4 pounds in the tube A cannot ſink 1 Inch, for example, but the pound in the tube B muſt mount 4 inches: Therefore the ratio of the force A, to the force B, being reciprocally as the ratio of the velocity of B, to the velocity of A, the weight A cannot at all raiſe the weight B, (by *Axiom.)*

2. If you put into the tube B, a different liquor from that in the tube A, it will ſtill remain in equilibrium, ſince the liquor in the tube B will weigh as much as that in the tube A, if it riſe in the tube B to the ſame height as in the tube A: For example, if into the place of the pound of liquor in the tube B, a pound of other liquor be pour'd, it will make an equilibrium with the 4 pounds in the tube A, to whatſoever height it riſes in the tube B.

COROL. I.

By this it appears, how ſmall a cylinder of water may counterbalance all the water in the ſea.

COROL. II.

If the body A be put into water, it will ſo far ſink as till it takes up a ſpace equal to a maſs of water of its own weight.

Demonſtrat. The body A, with the water under it, forms a cylinder, making an equilibrium with all the reſt of the water, as with another cylinder: Therefore (by *Cor.* preced.) this cylinder compoſed of the body A, and the water under it, ought not to weigh more than if it was all water, and did not riſe above the ſurface of the water; therefore the body A weighs as much as the maſs of water, whoſe place it occupies under the ſurface of the water. *Fig.* 14.

COROL. III.

In meaſuring the capacity of that part of a ſhip within the water we find its burden: For if, for example, we find the part of a ſhip which is under water to be equal to 1000 cubick feet, we conclude, that the ſhip, with all its lading, *&c.* is equal in weight to 1000 cubick feet of water.

PROP. IX.

Of ſuſpended Bodies.

LEMMA I.

Fig. 15.

If from any point, as G, of the line B G, the arch B D be deſcribed, and the right lines B D, B A, be drawn, and D F fall perpendicularly upon B A; laſtly, if A I be parallel to B D, I ſay, 1. That when the point F is between the points A and B, B D will have a leſs ratio to B F, than B A to A I.

Demonſt. The angle B D F being the complement of D B F, will be greater than the angle A B I, which together with the angle D B F makes but an acute angle. Imagining therefore the angle L D B equal to the angle A B I and the alternate interior angles B A I, A B D, being equal, (by 15. 1.) the triangles D B L, B A I, will be ſimilar; and conſequently (by 4. 6.) B A : A I :: D B : B L; but B D is leſs with regard to B F, than to B L, (by 8. 5.) therefore B D is leſs with reſpect to B F, than B A to A I.

Fig. 16.

2. If the point B fall between the point F, and the point A, B D will have a greater ratio to B F, than B A to A I.

Demonſt. The angle B D F being but the complement of D B F, or A B L, will be leſs than the angle A B I, which, with A B L, makes an obtuſe angle; for G B D is acute, (by 16. 3.) therefore the reſt will follow, as above.

LEMMA

LEMMA II.

If inſtead of BD, another line be drawn, as BC, and from the point C, the arch ED be deſcribed, and AH drawn parallel to BC, I ſay, 1. That when the point F falls between the points A and B, EB will have a leſs ratio to BF, than BA to AH. Draw the line EF. *Fig.* 15.

Demonſt. Since the alternate angles EBF, BAH, are equal, it only remains to prove, that ABH is leſs than BEF, (by preced.) which is evident: for if thro' the three points D, E, B, a circle be deſcribed, the point F will be found within it, ſince the angle BED is obtuſe, and BFD right; therefore the Angle FEB being ſubtended by a greater arch than BDF, will be greater, (by 22. 3.) but the angle BDF is greater than ABH, becauſe it is the complement of the angle ABD, which, with ABH, makes but an acute Angle; therefore the angle BEF will be greater than ABH.

2. If the point B falls between the point A and the point F, BE will have a greater ratio to BF, than BA to AH, becauſe the angle BEF is leſs than BDF. *Fig.* 16.

Demonſt. If a circle be deſcribed thro' the three points B, D, F, the point E will fall without the circle, becauſe the angle BFD is right, and the angle BED acute; therefore the angle BEF will be leſs than BDF, which being but the complement of DBF, will be leſs than ABI.

If the the weight B is ſuſpended by a chord, one end of which, as H is fixed, and the other end D, paſſing thro' the pully K, ſuſtains the weight D; I ſay, that when the weight B, ſhall be to the weight D, as the line of direction A B to A L, parallel to the line B N, that then the weights ſhall be in equilibrium.

Fig. 17.

Demonſtrat. As the weight D cannot raiſe the weight B, to the point G, (by *Ax*) becauſe its velocity B G, bears a leſs proportion to the velocity B E, of the weight B, (by *Lem.* 1. & 2.) than the force of the weight B, to the force of the weight D, ſo neither can the weight B, deſcend to the point I, (by *Ax.*) becauſe the velocity B F, of the weight D, bears a greater proportion to the velocity B R, of the weight B, than the force of the weight B, has to the force of the weight D, (by *Lem.* 1. & 2.) and conſequently they will remain in equilibrium, (by *Ax.*)

If the point H be not fix'd, but the chord paſſing thro' the pully ſuſtains the weight C, it will ſtill be the ſame thing; for the weight C will only ſerve to ſhorten the chord H B; ſo that we may conclude, that if the three weights D, B, C, are in equilibrium, the weight B will be to the weight D, as B A to A L, and to the weight C, as B A to A N, which is ſuppoſed parallel to H L; and conſequently the weight D, will be to the weight C, as A L to A N, or as the ſine of the angle A B L, to the ſine of the angle A B N.

PROP. X.

Of Acceleration,

Acceleration is the augmentation or increaſe of motion, cauſed by a continual application of the power.

		B			A		C
0.	1.	2.	3.	4.	5.	6.	7.
7.	7.	7.	7.	7.	7.	7.	7.

LEMMA.

The terms of an arithmetical progreſſion, beginning with 0, are only equal to the half of an equal number of terms, each equal to the laſt term of the progreſſion.

Demonſt. If you take two terms of the progreſſion, of which the one A, is as far diſtant from the laſt C, as the other B, from the firſt 0, their ſum will be equal but to the laſt term C; becauſe the term C exceeds the term A, juſt as the term B exceeds 0: Thus taking all the terms of the progreſſions two and two, each ſum will be equal to the laſt term C; therefore ſince the number of theſe ſums will be but half the number of the terms, conſequently the ſum of all the terms of the progreſſion will be but half the ſum of the ſame number of terms, each equal to the laſt term C.

1. If the weight A, falling in a free medium, run thro' the ſpace A B, during the firſt minute of its fall; during the ſecond, it will run thro' the ſpace B C, which is triple the ſpace A B.

Fig. 18.

Demonſt. Experience teaches us, that gravity continually increaſes the motion of heavy bodies, in their deſcent towards the center of the earth. Thus imagining the firſt minute divided into an infinite number of parts, we may conceive the motion of the weight A equally augmented thro' each of thoſe parts; and that having had no motion in the beginning of the minute, during the ſecond part of the time, it will have run thro' one part of the ſpace A B; then 2 in the third, 3 in the fourth, *&c.* We may further conceive, that throughout all the parts of the ſecond minute, it will have the ſame motion it had at the end of the firſt; and conſequently, if the weight A received no new motion during the ſecond minute, it would run thro' a ſpace double the ſpace A B; then ſince it does acquire a new motion, equal to that it got during the firſt minute, it will run thro' a ſpace triple to the ſpace A B.

2. In the ſame manner it may be proved, that during the third minute it will run thro' the ſpace C D, which is five times the ſpace A B; from whence we conclude, that the ſpaces run thro' will be in a duplicate ratio to the time of the deſcent, or as their ſquares.

3. The impreſſion made by one body upon another, is more or leſs according to the motion; that is to ſay, (according to *Def.* 3.) with regard either to the bulk of the body, or its velocity.

COROLLARY. I

If the weight A in its fall ſtrike a body at the point C, the impreſſion it will there make upon ſuch body, will be quadru- *Fig.* 18. ple what it would have been at the point B. Thus the impreſſions made by a body falling, increaſe in a duplicate ratio of the time of their deſcent, or as the ſpaces they run through.

COROL. II.

If in the room of a dead weight, we ſubſtitute a living power, as a man's hand A, ſtriking a mallet againſt the wedge B, *Fig.* 19. the ſame reaſoning will hold good, that is, the impreſſion of the mallet A on the wedge B, will be as the ſpace A B.

COROL. III.

It is not altogether the ſame thing with reſpect to ſprings, becauſe their force decreaſes in proportion as they approach their natural ſituation. Thus the impreſſion made *Fig.* 20. by the ſtring of a bow upon an arrow B, will not increaſe as the ſpaces B B.

COROL. IV.

Two bodies equal in every reſpect falling from the ſame height, will make the ſame impreſſion on the body they ſtrike; or if two unequal bodies fall from different heights, if the height of the firſt is to the height of the ſecond reciprocally as the weight of the ſecond to the weight of the firſt, their impreſſions will be likewiſe equal.

COROL. V.

One body ſimply laid upon another, will make the ſame impreſſion on it, as would another body falling from above, provided the acquired velocity of the one is to the ſimple motion of the other, reciprocally as the gravity of the firſt to the gravity of the ſecond. But as we know not the ſimple motion of heavy bodies, that is to ſay, the motion they would have if they were to deſcend without acceleration, we can determine nothing upon that point.

4. If a lever A C, without gravity, were ſo faſten'd at the point A, as that it might turn upon it, and being ſo raiſed as to form any angle with the horizontal line A B; if then a weight had been placed upon point as D, the lever will fall towards the horizontal line A B, with leſs velocity, than if the ſame weight had been applied at the point C, because the gravity of the lever will be the ſame in both caſes. *Fig.* 21.

COROLLARY.

If the weight be placed at the point D, it will not fall with ſo much force as if it were placed at the point C; nor will the impreſſion it makes upon the body upon which it falls be ſo great; and therefore it is, that the longer the handle of a hammer is, the greater is the force of the blow.

PROP.

PROP. XI.

The Application of theſe Principles to ſome Problems uſeful at Sea.

1. *With regard to Acceleration.*

Suppoſe the anchor A fixed in the bottom of the ſea; to the end of the buoy-rope A B, faſten a block B, thro' which rieve a rope, faſten'd at one end to the ſtern of the boat C D, ſo that the other end paſſing thro' the blocks C and D, (fix'd in the ſtern and ſtem of the boat, may be brought aboard the ſhip, which is likewiſe ſuppoſed to have an anchor out a-ſtern: It will then be eaſy to imagine, that when that part of the rope on board, is haul'd with due force, the boat will be drawn towards the anchor; and as the force of hauling is continued, the motion of the boat will be augmented; when therefore the boat comes a-pique the anchor, not being able to approach it nearer, it will draw the anchor with ſuch force as to raiſe it; and the cable being recovered, it may be brought aboard.

Fig. 22.

COROLLARY.

By this means an anchor may be weigh'd, when by reaſon of the enemies fire, a boat cannot be mann'd out with ſafety to recover it; it being in this caſe ſufficient only to ſend out a driver to fix the block, and rieve the rope.

2. *With reſpect to ſuſpended Bodies.*

1. Suppoſe the rope or tack A B, of which the end A, after paſſing round a block A, *Fig*. 23. is extended to the power F, which holds it, and the other end B, paſſing likewiſe thro' the block B, is faſten'd to the clue of the ſail E: Laying then hold of the rope at the point C, it may be eaſily drawn towards D, till the reſiſtance of the ſail is to the force of the power D, as the ſine of the angle C A D to the ſine of the angle C D A, (by *Prop*. 9.) without obliging the power F to give way, ſince the block A, againſt which the rope A B bears hard, being immoveable, will not let it run. When therefore the power D returns to the point C, the power F will recover part of the rope A D B, eſpecially if the power D oppoſes itſelf to the power E, in drawing the rope towards A. Thus after ſeveral attempts the tack may be belayed as it ought.

2. Suppoſe the cable AB, one end of which is fix'd in the veſſel A, and the other *Fig*. 24. faſten'd to the anchor B; ſuppoſe another ſhorter cable B D: It is evident, that theſe two cables being ſuſpended between the two points to which they are fix'd, the cable A B will bear hardeſt upon both points, and alſo with reſpect to itſelf, as being heavieſt: Notwithſtanding which, the anchor B will ſtrain leſs upon the cable A B, than upon the cable B D; and the cable A B will be leſs ſubject to break, or part, as the ſeamen phraſe it; 1. Becauſe the cable A B being longer, will form a more concave figure, (by *Prop*. 9.) great part thereof will reſt upon the

ground near the anchor; upon which therefore it will have leſs ſtrain, and particularly will not raiſe up its ring, as the cable B D, which renders it more liable to break its hold. 2. Becauſe when the ſwell of the ſea raiſes the ſhip's head, as from the point A to the point C, or from the point D to the point E, its effect is plainly greater upon the cable B D, than upon the cable B A.

3. *With reſpect to Liquors.*

1. If an unladen ſhip A, be aground in a haven, ſhe may eaſily be floated, by placing on each ſide a lighter or barge, ſo loaden that their gunnels be brought down almoſt to the ſurface of the water, then fixing beams in the ports of the ſhip, let them reſt upon the ſides of the lighters, extending quite over on both ſides, then lightening the lighters, as they riſe, they will lift up the ſhip. *Fig.* 25.

2. The ſame thing may be done with large veſſels fill'd with water, which being faſten'd to the ſides of the ſhip, the water being pump'd out will in like manner raiſe the ſhip, as the veſſels begin to float upon the water.

4. *With reſpect to Cranes and Pullies.*

The uſe of Cranes and Pullies is ſo common, that I ſhall not need to dwell much upon it, I ſhall therefore juſt obſerve, that what obſtructs the force of capſtons when the ſhip or boat is brought a-pique the anchor, not only proceeds from the reſiſtance of the anchor itſelf before it quits its hold, but alſo from the rubbing of the cable in the hawſe. When therefore the anchor's hold ſeems too great for the power of the capſton, fix a rope (or meſſenger, as the ſeamen *Fig.* 26. term it) upon ſome part of the cable, as B, and after having brought a large tackle upon it, bring the fall of the tackle to the capſton, which then will raiſe the anchor without difficulty, not only as the tackle will have a better purchaſe, and will conſequently augment the force, but becauſe the rubbing of the cable in the hawſe will likewiſe be thereby prevented.

REMARK.

Sometimes an anchor is ſo fix'd or buried in the ground, that nothing is able to raiſe it. In ſuch caſe, if the tides are conſiderable, they let the ſhip ride with her cable as much as poſſible a-pique during the ebb, and then as the ſhip riſes with the flood, either the anchor is raiſed, or the cable breaks.

5. *With reſpect to Levers.*

To raiſe a yard or maſt, as A B, to place it in a ſhip, it is moſt convenient to faſten *Fig.* 27. the purchaſe at the end A; for if it be ſeiz'd by the middle C, the reſiſtance will be found twice as great, (by *Prop.* 2.) and if it be ſeiz'd about the end B, which is ſtill farther diſtant from the place it is to be drawn to, a ſtill greater force muſt be applied.

FORTIFICATIONS.

TO *fortify* a Place, is to put it into a condition of being defended by a few men againſt the attack of a much greater number. As the ſeveral Rules laid down in this Tract are all conformable with this definition, ſo the following Axioms may be look'd upon as immediate conſequences thereof.

AXIOMS.

1. The *Fortifications* of a place ought to be as little extended as poſſible.

2. They ought to be as uniform as poſſible.

3. They ought to be ſo contrived as at once to cover the defendants, and diſcover the approaches of the enemy, and adapted to the common weapons of defence.

4. They ought, as much as poſſible, to be ſuch as are eaſy to build, hard to deſtroy, and not difficult to be repaired.

5. Each part ſhould be defended by as many places as may be, and liable to be attack'd from as few places as poſſible.

DEFINITIONS.

Fig. 1. 1. The *Rampart* is a bank of earth, encompaſſing the place within the walls, as E B E.

2. A *Baſtion* is an advanced part of the rampart, in form of a Pentagon, as B.

3. The *Curtain* is that part of the rampart lying between two baſtions, as E.

4. The *Moat* is a large ditch encompaſſing the place without the walls, uſually fill'd with water, as H F, H F.

5. The *Cover'd Way*, F F F, is a paſſage along the border of the ditch oppoſite to the place, and is covered with a bank of earth falling ſlopeways towards the country, till it comes to the level of the ground. This ſlope is call'd the *Glacis*, as G, G, G.

6. The *Profile* of a Fortification is the ſection made by a plane cutting it from the middle A, to the extremity of the glacis K. There are commonly two made, one paſſing thro' the middle of the curtain, the other thro' the middle of the baſtion,

Fig. 2.

7. In the Profile A R, ſuppoſed to paſs thro' he middle of the curtain, there are o be conſider'd, 1. The *Place of arms* M, which is a ſtreet or void ſpace between the houſes and the rampart. 2. The *Talus* r Slope of the rampart B C. 3. The *Rampart* CD. 4. The *Banquet* or little bank D. 5. The pper ſurface, of which E is call'd the *Parapet*. . The Sone face or front of the wall V. 7. The *carpe*, or ſteep bank of the ditch next the town V R. 8. The *Moat* or ditch R L. 9. The *hannel* of the moat T. 10. The *Counterſcarp*, r that bank of the ditch oppoſite to the place L. 1. The *Cover'd Way* L. 12. The *Paliſade* N. 3. The *Glacis* of the counterſcarp M.

Fig. 1.

8. The lines F g being prolonged to the curtain, re call'd *lines of defence*; and the ngle g H g, which they form, is call'd he *flanking angle*. The planes F g of he baſtion are call'd its *faces*; the planes a g its *lanks*; and the planes a B its *demigorges*. The ngle of the faces g F g is the *flanked angle*; the ngle g a a, is the *angle of the flank*; the angle g F, the *angle of the epaul* or *ſhoulder*; the ngle a B a, the *angle of the gorge*; the angle B A B, he *angle of the center*.

PROP. I.

he manner of laying down upon paper the Plan of a regular Fortification.

Regular Fortifications are ſuch as are made pon a regular Polygon.

1. Meaſure exactly the circuit of the place t be fortified, at about 12 paces diſtant from th houſes, and divide the whole circuit by 150 geometrical paces at leaſt, or by 180 at moſt, and th quotient will give the number of baſtions, in ſuch manner that their lines of defence ſhall not excee the carriage of a muſquet.

Fig. 3.

2. Inſcribe in a circle, a polygon with as man ſides as the place is to have baſtions; and from the center A, thro' the angl B of the polygon, draw lines at pleaſure, which lines are call'd *Principals.* Afterwards take B c equal to $\frac{1}{5}$ of the ſide of the polygon, and B D equal to $\frac{1}{8}$, then draw the lines o defence D c, and from each point c, raiſe perpendiculars, which meeting the lines of defence in th points E, will form the baſtions c E D E c.

This method ſeems preferable to all others, a it is the moſt ſimple, and makes the baſtions wel proportioned: It avoids forming the flanked angle too acute, or too obtuſe, and gives the flank al the extent it is capable of.

3. Having thus deſcribed the outward circui of the rampart, draw from the angles of the ſhoulder E the lines E M, parrallel to the faces D E, which will meet each other in the principal line at the points M, and before the curtains in th points G, and ſo will determine the outward circuit of the moat M, M, M, which ought to b rounded before the angles D, by an arch deſcribe from the point D, having for its radius the perpendiculars D y.

4. To finiſh the plan, draw within the place lines parallel to thoſe which form the outward circuit of the rampart: 1. At the diſtance of $\frac{1}{5}$ of the flank, for the parapets. 2. At the diſtance of $\frac{1}{2}$ the demigorge, for the ramparts. 3. At the diſtance of 5 paces from the rampart, for the place of arms. 4. At the diſtance of 16 foot within the rampart, for its talus or ſlope. 5. At the diſtance of 5 foot, from the parapet, for the banquet. In the ſame manner, on the outſide of the moat, muſt be drawn lines parallel to its outward circuit: 1. At the diſtance of $\frac{1}{4}$ of the flank, for the cover'd way. 2. At the diſtance of $\frac{2}{3}$ of the flank, for the glacis.

5. It will be eaſy to lay down the profile, and to aſſign the ſeveral heights: For ſuppoſing A R to repreſent the level of the plane; take A B, 5 paces, for the place of arms B A, and the perpendicular O C, 16 foot, for the talus or ſlope B C; the thickneſs of the lower part of the rampart B R, 12 paces, the upper part C D to the banquet, 6 paces $\frac{2}{3}$; the banquet 6 foot; the thickneſs of the lower part of the parapet D V, 3 paces $\frac{1}{3}$; the upper part 2 paces $\frac{1}{3}$; its inward height E D 6 foot; its outward height E V, 5 foot; the talus of the rampart G R to the moat, 7 foot; and the talus of the ſcarp x y, 2 paces; the depth of the moat x R, let be 16 foot; the width of its channel T, 15 foot; the talus of the counterſcarp a K, 10 foot; the banquet of the cover'd way, 5 foot.

Fig. 2.

REMARKS.

I.

Formerly it was an uſual practice to raiſe at the foot of the ſcarp a bank of 30 or 40 foot broad, ſhelter'd with a parapet of 15 foot thick, and 6 foot high, which was call'd the *fauſbraie*; but as ſuch place is too much expoſed to be of any great ſervice, it is not now uſed.

II.

As the flanks are the grand defence of Fortifications, they are ſometimes cover'd with an *oreillon*, or an *epaulement*, after different manners.

Fig. 4.

1. Divide the flank B A equally in the point E, and raiſe the perpendicular E D $\frac{1}{3}$ of the flank; then draw D C parallel to the flank, till it meet the face prolong'd in the point C, ſo ſhall you have the *epaulement* B C D E. But if you would have an *oreillon*, from the point F, where the line E D meets with the face, with the diſtance F C, deſcribe the arch D C; and from the middle of that arch G, with the diſtance G C, deſcribe an arch which will form the oreillion BCDE.

Fig. 5.

2. Mr. *Vauban*'s flanks may likewiſe be very nearly imitated, thus; Take B C $\frac{1}{3}$ of the flank, and from the point C draw F C equal to B C, in ſuch manner that being prolong'd it will meet the flank'd angle of the next baſtion: Farther from the point E, where the face prolong'd meets with it, with the diſtance E F deſcribe an arch F G, the midde of

which H, will give you the center of the arch B C. Laſtly, from the point D, the middle of the line C A, with the diſtance D F, deſcribe the arch F K, which will meet the line of defence in the point K, and draw K A, which will finiſh the flank A K F C B G.

III.

I ſay nothing of *Caſemates*, which are now in a manner out of uſe.

PROP. II.

To lay down Outworks upon paper.

Under the name of *Outworks* is here comprehended every kind of work detached or ſeparated from the rampart of the place, advancing farther towards the campaign.

1. To raiſe a *ravelin* before the curtain: From the middle of the curtain A, draw an indeterminate perpendicular, upon which ſet off A E equal to the curtain, and from the point E draw lines to the angles of the ſhoulder, which cutting the outward bank of the moat in the points C, will give the outward circuit of the ravelin E C B C E. As to the thickneſs of its parapet, its cover'd way, and its glacis, they take their proportions from thoſe of the place itſelf; but the width and depth of the ditch are but of half the dimenſions allow'd to the grand moat of the place; which obſervation is general with reſpect to all ſorts of outworks. *Fig.* 6.

2. To

Fig. 7.

2. To erect an *half-moon* before the flank'd angle: Take upon the principal line from the point o, where it cuts the out-ſide of the moat o B, equal to $\frac{2}{3}$ of the face; then having drawn, from the point B, lines to the flanking angle G, produce the faces F A, till they cut the bank of the moat in the points D, and the lines B G in the point E, and you will have the exterior circuit of the half-moon o D E B E D.

Fig. 8, 9, 10.

Fig. 8, 9.

3. For *Horn-works:* From the middle of the curtain A, draw the perpendicular A B, equal to the ſide of the polygon N N, and from the angles of the flank G, draw G C parallels, and equal to the line A B, they will cut the outward bank of the moat in the points D; then take B E equal to $\frac{1}{4}$ of the curtain, and draw C E, which will finiſh the horn-work D C E C D.

Fig. 10.

Or if you like it better, taking C E $\frac{1}{3}$ of C C, and C F $\frac{1}{4}$ of C C, draw the perpendiculars E H upon the line F F, and they will cut the lines of defence C H in the points O, and ſo will give the circuit of the horn-work D F C O H H O C F D.

Or laſtly, from the point B, drawing the perpendiculars B C equal to half the curtain, the lines A C will cut the outward bank of the moat of the half-moon in the points D; then having taken B O, and B E, $\frac{1}{4}$ of C C, and having drawn the lines C E, from the middle of them G, draw G O, and you will have the outward circuit of a ſwallow's tail CDGOGDC.

4. For *Crown-works:* 1. From the angle of the polygon A, upon the principal line take A B, equal to $\frac{2}{3}$ the ſide of the polygon; then making B C on both ſides equal, and parallel to the ſides of the polygon, draw from the angles of the flank E the lines E C, which will cut the outward bank of the moat in the points D. Farther, make two demi-baſtions upon the lines, on each ſide one, after the manner laid down in the ſecond manner of deſcribing a horn-work, (*vide Fig.* 9.) and you will have the exterior circuit of the crown-work D C G H H G B G H H G C D. *Fig.* 11.

2. The ſame thing may be done in drawing A B from the middle of the curtain, and making the angle C B C equal to the angle of the polygon. *Fig.* 12.

Prop. III.

The Demonſtration of the foregoing Rules.

To demonſtrate the foregoing rules, let them be applied to ſeveral polygons, and it will appear that they will always produce Fortifications conformable to the axioms at firſt laid down, more eſpecially the following faults will conſtantly be avoided.

1. It is a great miſtake in Fortifications, when the lines of defence A B are two long, becauſe the enemy may then with ſafety approach the point B, where they will be in ſecurity, at leaſt, from muſket-ſhot, till they *Fig.* 13.

have time to ſhelter themſelves. Now, by the foregoing rules, the greateſt length of the ſaid lines of defence is but 180 paces.

2. When the flanked angle A B C is too acute, the enemy's cannon will eaſily make a *Fig.* 14. breach; for the point D being only ſuſtain'd by the ſmall thickneſs D E, the ball will quickly ſhake it; whereas in a baſtion leſs acute, as A B F, the point D is ſuſtain'd with the whole thickneſs D G. Our method therefore gives the flank'd angle as little acute as poſſible, nay, very often obtuſe.

3. It is true, the flank'd angle ought not to be ſo obtuſe as to make the baſtion too *Fig.* 15. ſtrait, as A B C; but in other reſpects they are the more ſubſtantial, and better, provided the ſaid inconvenience be avoided, as we have done in fixing the length of the capital B D.

4. The main defence being in the flanks, they ought not to be too ſhort; one method makes them longer than commonly they are laid down, except in the ſquare, where large flanks would render the flank'd angle too acute.

5. It is true, our method excludes ſecond flanks A B, ſo much recommended by ſome *Fig.* 16. *Dutch* Engineers, that the face of the baſtion C D may not only be defended by the fire from the flank B C, but likewiſe from the ſecond flank A B. Neverthleſs, it appears to me, that the defence from the ſecond flank, where the guns muſt be pointed ſlopewiſe over a

very thick parapet, cannot be very conſiderable; and I ſhould rather chuſe to enlarge the flank B C, by the length C E, than to have recourſe to a ſecond flank A B.

Prop. IV.

Some other Methods.

I.

Some Engineers (as *De Ville*) divide the ſid A A of the polygon into 6 equal parts, and make the demigorge A B one of thoſe parts, as alſo the flank B C; then for places which have leſs then 6 baſtions, they draw the line of defence D C E, which gives the flank'd angle C D C. For other places they deſcribe the ſemicircle upon the line C C, which cutting the principal in the point D, gives the flank'd angle C D C. This method, in my opinion, makes the baſtion too ſmall, nor can I ſee why they ſhould prefer right angles to obtuſe ones, unleſs it be for the ſake of a ſecond flank, which I have before obſerved to be of little advantage.

Fig. 17.

Fig. 18.

II.

Others (as *Pagan*) begin with the exterior ſide of the polygon A A, which they divide equally in the point B; then they draw the perpendicular B C, which they extend to 30 fathoms, and after having drawn the indeterminate lines A C E, they ſet off A D 30 fathoms, and from the point D, they draw the flanks D E perpendicular upon the lines of defence E A; and laſtly, they draw the cur-

Fig. 19.

tain E E. They think, that the flanks being perpendicular upon the lines of defence, they are moſt advantageouſly placed, their fire being thereby render'd more large, and more direct: Nevertheleſs, I cannot prefer theſe ſort of flanks to thoſe commonly uſed, becauſe they are more expoſed to the enemies batteries, without any poſſibility of ſhelter.

III.

There are ſome Engineers who give $\frac{1}{4}$ of the ſide of the polygon to the demigorge; but by this means the flank, and conſequently the defence, becomes much leſſen'd: However I do not reject this manner in polygons above the decagon.

IV.

Others, to make amends for what is wanting in flank by the foregoing method, make a flank in the middle of the face, and ſometimes two; but the face is thereby render'd much the weaker. *Fig.* 20. *Fig.* 21.

V.

There are alſo ſome who would have the baſtions detach'd from the place, and ſeparated by a ditch, the more eaſily to obſtruct the progreſs of the enemy after he has taken the baſtion. *Fig.* 22.

VI.

Others are for making the baſtions of the place extreamly ſmall, and then ſurrounding them with other detach'd baſtions, which ought nevertheleſs to be commanded by the firſt. *Fig.* 23.

PROP.

PROP. V.

To compute or calculate Fortifications.

It is of great importance, after having traced out Fortifications upon paper, to bring them to a computation, that is, to find the length of all the lines and the value of all the angles, which will afford much help to the laying them down afterwards upon the ground.

1. For the greater exactneſs, we begin by multiplying the ſide D D of the polygon by 100; then cutting off the two laſt figures of every number found, the two figures ſo cut off, with the denominator 100, will form a fraction.

Fig. 24.

2. In the triangle A D D the angle A may be known, by dividing 360 by the number of ſides in the polygon; then dividing the ſupplement of the angle A into two equal parts, you will find the two angles D, which are equal; after which ſay, (by *Trig* 13.) As the ſine of the angle A : to the ſide D D : : ſo is the ſine of the angle D : to the radius A D.

3. In the triangle D E B, the angle D being known, (by the preceeding, as alſo the ſide B E, which is $\frac{4}{5}$ of the ſide D D, (by conſtruct.) and the ſide D B, which is $\frac{1}{3}$ of it, the whole triangle D E B may be reſolved, (by *Trig.* 13.) by which means will be found both the angle B, which is $\frac{1}{2}$ the flanked angle, and the line of defence B E, as alſo the angle E.

4. In the triangle E C E, rectangular at the point E of the line C E, (by conſtruct.) the ſide E E being known to be $\frac{3}{5}$ of D D, and the angle E to be right; you may reſolve the triangle E C E (by *Trig.* 12.) by which you will know the flank C E, and the line C E oppoſite to the right angle, which being ſubtracted from the line B E, will give the face B C: You will likewiſe by the ſame means know the angle of the ſhoulder E C B.

5. The foregoing rules may alſo be eaſily applied to outworks. The following calculation is an example of an heptagon, whoſe ſide is ſuppoſed equal to 170 fathoms.

The computation of an Heptagon.

Lines.	*Fathoms*
Inward ſide ——	170
Radius — —	196
Demigorge —	34
Capital — —	57
Curtain — —	102
Line of defence —	168
Flank — ——	32
Face —— —	61
Outward ſide —	218

Angles.	*Deg.*	*Min.*
Angle of the center	51	25
Angle of the Polygon, or of the gorge	128	34
The flanked angle	93	36
The flanking angle	145	2
The angle of the flank	107	28

PROP.

PROP. VI.

To trace out irregular Fortifications upon paper.

Irregular Fortifications are ſuch as are raiſed about irregular polygons: But in theſe irregular works, we muſt follow, as much as poſſible, the proportions laid down for the regular ones.

SECT. I.

Of irregular Baſtions.

1. As Baſtions ought to be ſo built that their cannon may ſweep the ſurface of the circumjacent campaign, if they are found too high for that purpoſe, let a ſecond baſtion be added beneath the firſt, or even a third, if it is neceſſary, ſtill taking care that their faces be well defended. But if a baſtion is too low, raiſe a cavalier upon it. A cavalier is a maſs of earth, well beat, of about 26 fathoms radius, whoſe center is commonly placed in that point of the baſtion where the defences meet; it is commonly made two fathoms high, and with a talus of 7 foot; it ought alſo to have a parapet of 3 paces broad, and 6 foot high; it may be made of any figure, but moſt commonly is form'd as the baſtion.

Fig. 25.

It will ſometimes happen, that the faces of a baſtion would become exceſſive long if they were to be extended till they meet; in ſuch caſe, they are uſually cloſed with a returning angle, the manner of fortifying which ſhall be hereafter laid down.

Fig. 26.

3. If one ſide of the polygon, as A B,
Fig. 27. is too long, it may be remedied ſeveral ways: 1. If it be long enough to admit
Fig. 28. a baſtion in the middle, let one be raiſed, as C. 2. If it is not long enough to admit ſuch baſtion, it will be proper to form a ſalliant angle, as A C B. 3. Or the
Fig. 29. baſtions on each ſide may be advanced nearer each other, and both intirely form'd upon the line A B.

SECT. II.

Of returning Angles.

They ought to be avoided as much as poſſible; but if the ground obliges you to have recourſe to them, they may be fortified after one or other of the following methods:

1. Suppoſe the returning angle D C D, which is not obtuſe; take C E and C E, each
Fig. 30. of 8 or 10 paces, and having drawn the perpendiculars E F of the ſame length with the lines F G, you will have the exterior circuit of the tenail work D E F G C
Fig. 31. G F E D; to which may be added an oreillion.

2. Inſtead of the faces G C, may be
Fig. 31. drawn G G: Or, from the point C, with the diſtance C E, deſcribe the arch
Fig. 32. E G G E, in order to raiſe a work of ſtrength.

3. If the angle D C D is obtuſe, take C G of 25 fathoms, and of the ſame length draw the perpendiculars C H; ſo ſhall the line H H finiſh the outward circuit of the platform D G H H G D. There may be alſo added oreillions or epaulements upon the flanks G H. *Fig.* 33.

SECT. III.

Of Redans.

When a long extended ſide of a place cannot admit of baſtions, by reaſon it is cut off by a river, or ſtands upon a ſteep aſcent, it may be fortified with Redans. Suppoſe, for example, the ſide of a place A B; divide it into parts of 40 paces or fathoms each in the points C; then draw the perpendiculars C D of 4 or 5 paces, and the oblique lines C D will finiſh the exterior circuit of the redans A D C D C D B, to which may likewiſe be added oreillions. *Fig.* 34.

SECT. IV.

Of Triangles.

There is ſeldom any occaſion of fortifying Triangles; however, the beſt way of doing it, is with baſtions, with returning angles. *Fig.* 35.

SECT. V.

Of Citadels.

1. The Forifications of Citadels are of leſs extent than thoſe of the towns themſelves, the ſide of their polygon ſeldom exceeding 80 geometrical paces.

2. They ought intirely to command the places they are intended to defend; ſo that the place itſelf ſhould lie open on the ſide next the citadel.

SECT. VI.

Of Sconces.

Sconces are little forts, which ſerve to defend camps, the lines in a ſiege, batteries, bridges, &c.

1. Redoubts are ſquare works, one angle of which points towards the enemy; they ought to be about 40 paces in circumference; their rampart 16 foot thick at the bottom; their talus and their height 5 foot; their parapet 5 foot high, and 8 thick; the whole ought to be compos'd of earth well ramm'd, and faſcines; their ditch ſhould be 18 foot wide, and 6 deep.

2. Star Forts are thus made; ſuppoſe the radius of the polygon A A to be 110 or *Fig.* 36. 115 paces, divide each ſide by the perpendiculars B C, then making B D one third of B C, draw the lines A D, and you

will have the exterior circuit of the little fort B A D, for the reſt is much the ſame as in redoubts.

SECT. VII.

Of old Places, Caſtles, Bridges and Camps.

1. As to places fortified after the antient manner, all that can be done, is to add ſome out-works, as ravelins and horn-works, before the towers and the curtains.

2. Caſtles are encompaſs'd with a moat and a fauſbras of earth well beat, with a kind of baſtions, whoſe magnitude muſt be determin'd by that of the caſtle.

3. Camps are alſo fortified with ditches, the earth thrown out of which ſerving to make a kind of rampart, with a parapet circumſcribing the camp, and caſt up in ſuch a figure as may beſt defend the ſeveral parts of the rampart; redoubts, and ravelins, *&c.* are alſo commonly added.

4. Bridges are fortified with ravelins, and crown-works, extending from each ſide to the river. In one word, all other poſts which are to be fortified with expedition, muſt be ſecured with ſuch kind of works; care being always taken that every ſide be well defended.

PROP. VII.

Of Marine Fortifications.

Though Marine Fortifications have nothing peculiar in them, yet it may not be improper to give the following directions with relation to batteries.

I.

In raiſing batteries to hinder a deſcent, care ſhould be taken to diſpoſe them in ſuch places where the deſcent is moſt eaſy, as in places where the ſea has a clear bottom, the road large and ſafe, and the ſhore not ſteep, the guns upon theſe batteries ſhould be ſo levell'd, as to ſcour the ſurface of the water, and brought as near as poſſible to the edge of the ſhore, that they may fire effectually upon the ſmall boats as they approach; yet ought the battery to be ſo elevated, as to diſcover the enemy at a diſtance.

II.

It is likewiſe convenient to have batteries to play upon ſuch places where there is good anchorage, and theſe batteries ſhould be ſomewhat more elevated, becauſe a cannon, which commands the deck of a veſſel, is more terrible to the crew; and beſides, a bullet which enters a ſhip from above downwards, puts her into the greateſt danger of ſinking.

III.

It is alſo neceſſary to erect batteries at the entrance of roads; but they muſt be ſo made, as to diſcover ſhips at a great diſtance; ſome of their guns ought to be very much elevated; but at the ſame time they ſhould have others rais'd only 12 or 15 foot above the level of the ground, to fire upon ſhips as they approach the ſhore.

IV.

It is of the utmoſt importance to plant batteries which may command the anchorage of roads and ports, to the end that the enemies ſhips, after having paſs'd the fire of the entry, may not remain in ſafety. Upon extraordinary occaſions, they ſometime ſink ſhips in the middle of roads, and raiſe ſeveral batteries at proper diſtances. The ſame thing may be done at the entry of roads.

V.

It is very neceſſary that theſe ſorts of batteries ſhould be defended by ſome works againſt attacks, and ſhould, if poſſible, be under the fire of the place, at leaſt they ought not to be too far advanc'd.

REMARK.

I ſay nothing of the manner of beſieging or defending a place, or of the drawing up an army, theſe being things which depend more on experience and good ſenſe, than on mathematicks.

PROP. VIII.

To trace out Fortifications upon the Ground.

1. Having exactly conſider'd all the parts of the plan, mark the point where you ought to begin, by there fixing a piquet.

2. If the determin'd point be the center of the polygon A, extend the cord A B to the *Fig.* 37. length preſcrib'd in the plan; give alſo to the ſides B B their proper length, and that will determine the angles, and form the polygon.

3. If the determin'd point B is the angle of the polygon, from the point B extend the cord B F, ſo that the line B F may tend directly to the center of the polygon, then extending the cord B B to the length requir'd for the ſide of the polygon, by the help of your inſtrument, ſet off the angle F B B as the plan directs; in the ſame manner muſt be laid down the ſide B G of the polygon, according to its requir'd length, and ſetting off the angle F B G as before, *&c.* If in proceeding thus with each ſide of the polygon, the laſt cloſes the figure, the opperation is right; but if it does not cloſe, you may eaſily correct the deficiency.

4. The polygon being thus traced out, ſet off the demigorges B C, and the capital lines B D, and having drawn the perpendicular C E for the flanks, take notice if the points D E C are in the ſame line, which will ſerve to correct any error that may have happen'd; thus will the baſtions C E D E C be exactly traced out.

5. Purſuing the ſame method, you will eaſily accompliſh the whole plan, ſtill ſerving yourſelf with the principal lines and perpendiculars drawn by the middle of the curtain.

PROP.

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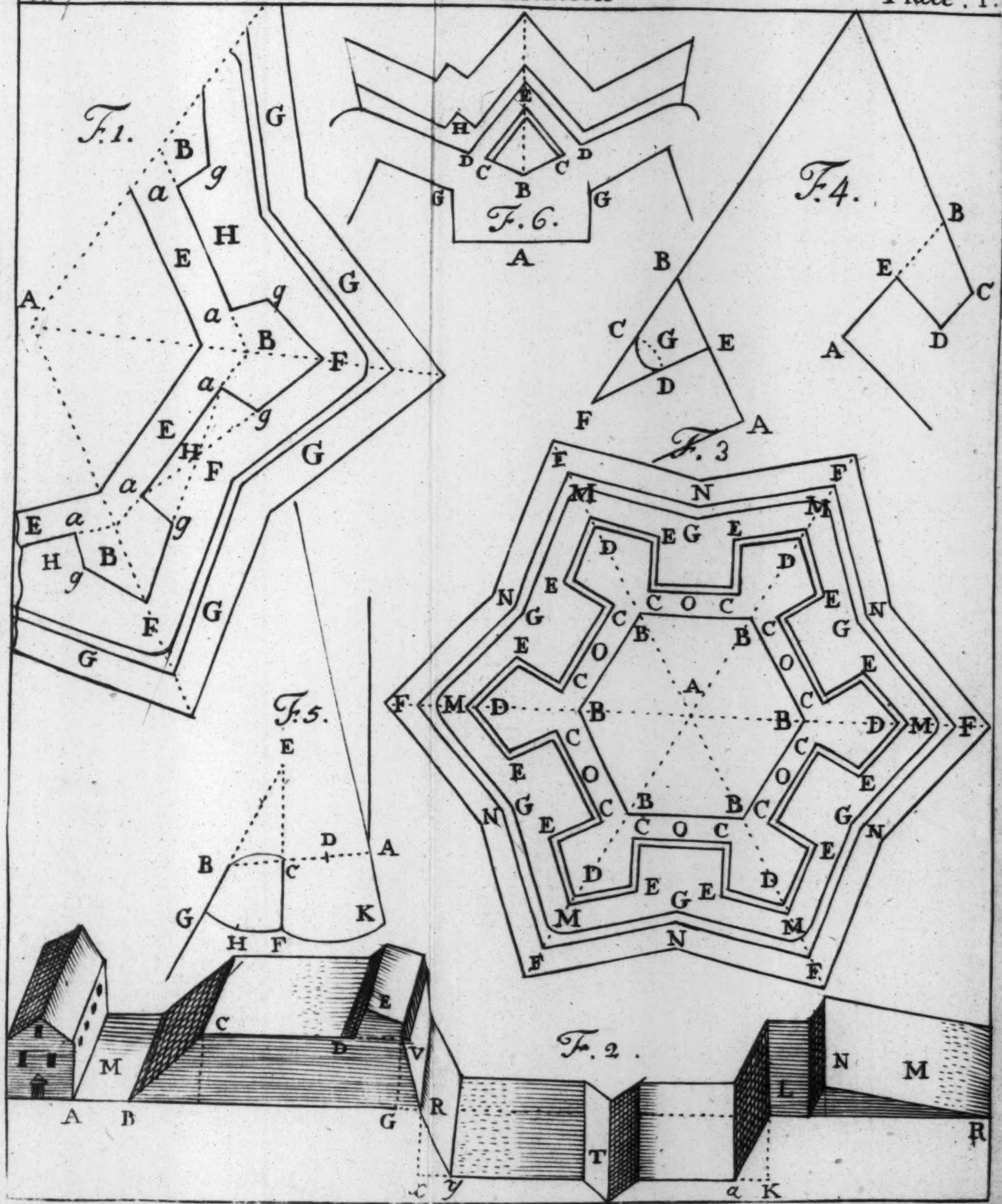
Fortification Plate . 1.



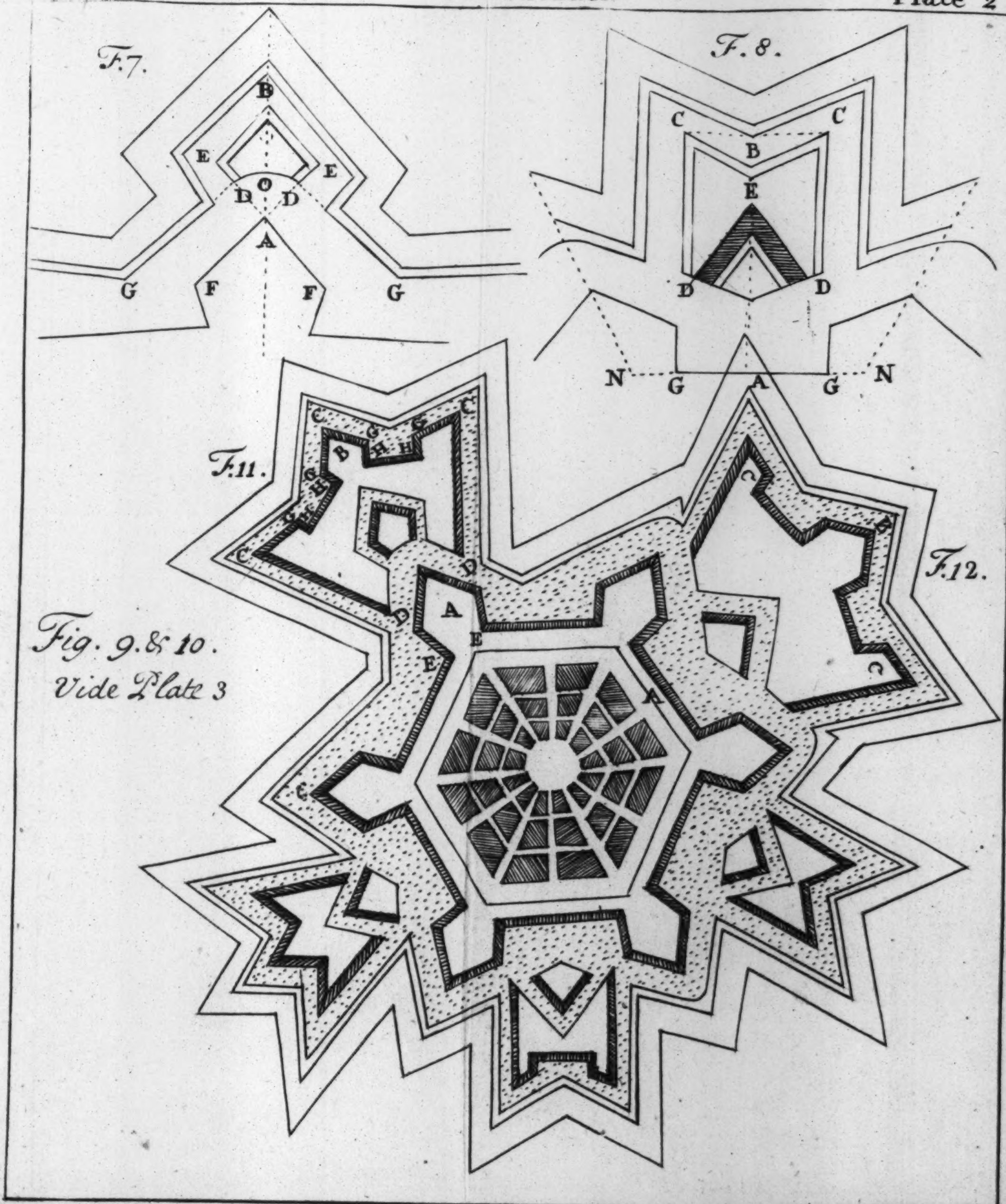
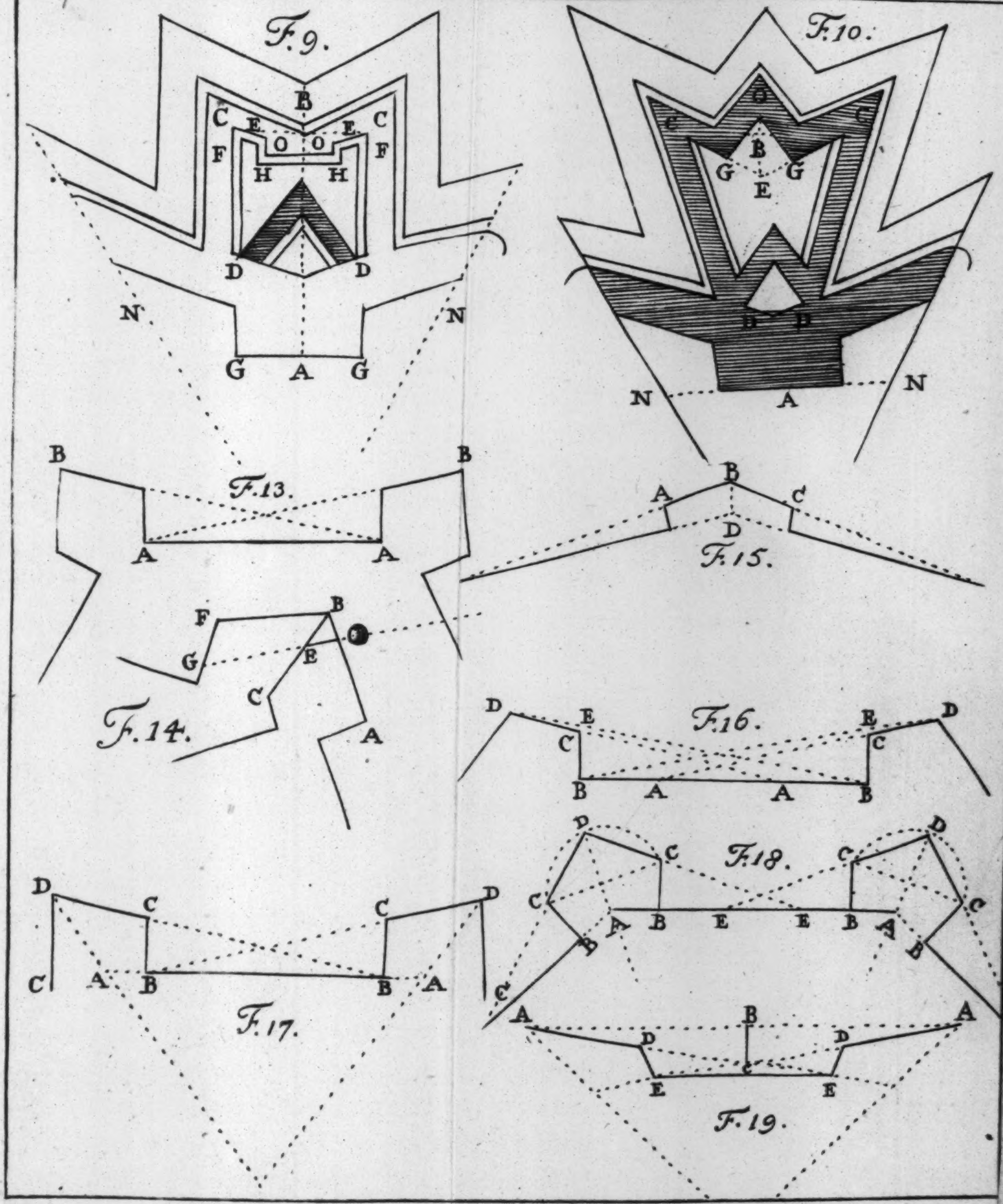


Fig. 9.& 10.
Vide Plate 3

Fortification

Plate . 3.



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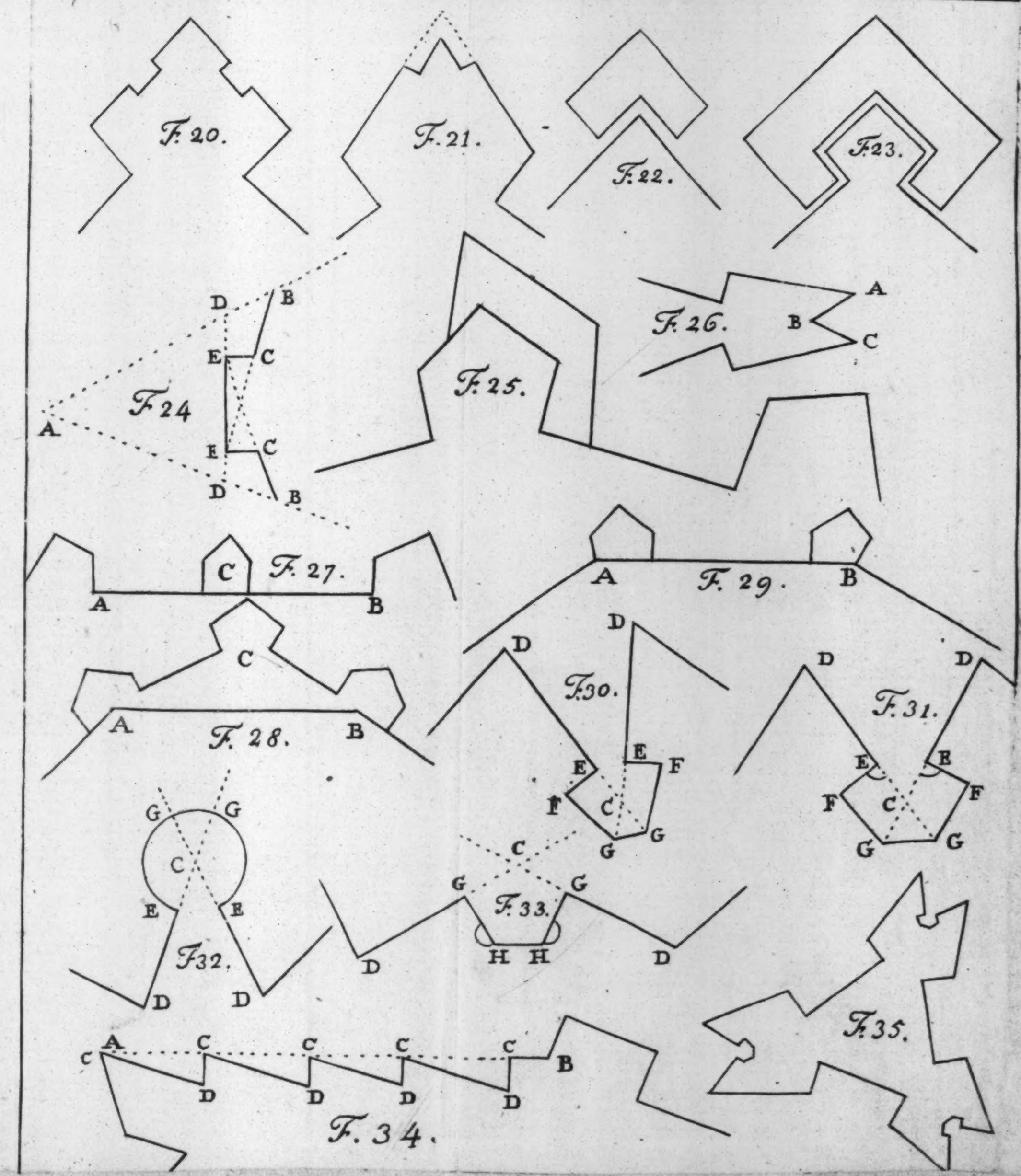
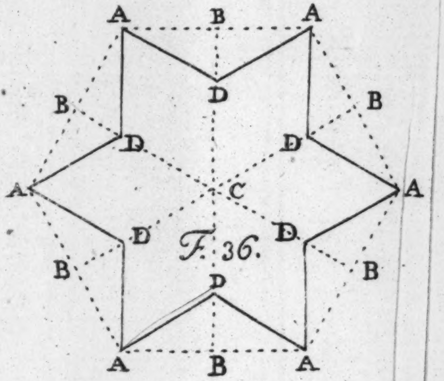
Fortification Plate . 4.





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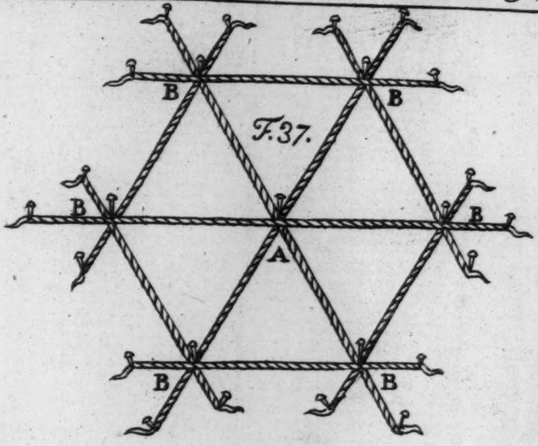
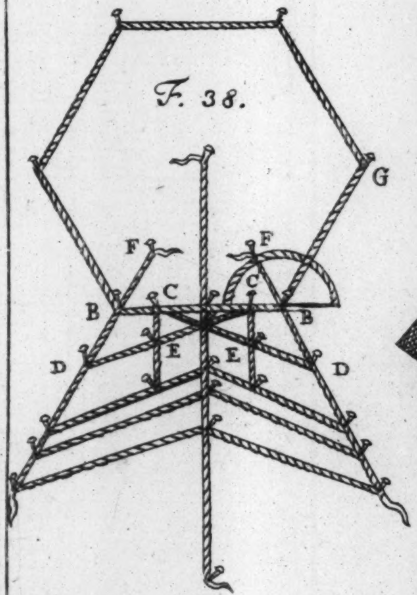


F. 37.

F. 38.



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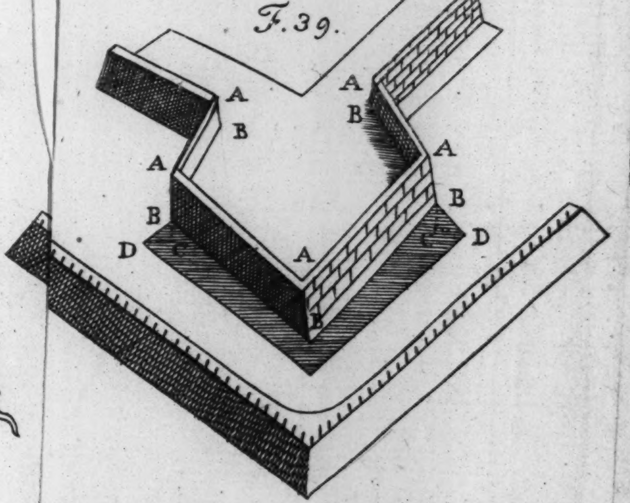

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PROP. IX.

To put your Plan in Perſpective Orthographically.

Fig. 39.

1. Having laid down your plan with all the lines which ought to mark the thickneſs, draw from each angle A, the vertical lines A B of ſuch length as each point A ought to appear rais'd above the level of the ground, then drawing the lines B B, and obſerving all the lines A B and B B, which ought to appear, that is to ſay, which are not loſt in any thickneſs, or hid behind any bod , ſo will your plan appear rais'd.

2. If you would expreſs the talus, take the vertical lines B C of the length of the talus's height, and draw the line C D of the length of the talus, and from the moſt convenient ſide, and the lines B D ſhall expreſs the talus.

3. To finiſh the perſpective, we muſt ſuppoſe the light to come from one ſide, and leaving the faces oppoſite thereto without ſhade, give a greater or leſſer ſhadowing to the other faces, according as they more or leſs are diſtant from the ſide whereon the light directly falls.

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GUNNERY.

IT is not the deſign of this Tract to offer every thing neceſſary to be known concerning Artillery; but only ſuch things in that Art, as have a more particular dependence on the Mathematicks.

DEFINITIONS.

1. Suppoſe a cannon and its profile to be A B, its cavity or chaſe will be B L, its mouth or muzzle B, its breech or coyle A, its platband E, its vent or touch-hole L, its firſt reinforcement I, its ſecond reinforcement F, its trunnions G, its dolphins H, its vacant cylinder G B, its aſtragal D, its neck C, its face C B. *Fig.* 1.

2. The calibre or bore of a cannon is the bigneſs of its mouth with reſpect to the ball or bullet it may receive.

3. Cannons are diſtinguiſhed, with regard to their length and bore, into three kinds, *viz.* a *Cannon*, a *Culverine*, and a *Baſtard Cannon.*

The cannon has a length proportioned to its bore; the culverine has the ſame length, but its bore is but of half the diameter of the bore of the cannon; the baſtard cannon has alſo the ſame length, but the diameter of its bore is a mean between that of a cannon and a culverine.

4. The faulcon and faulconet, are ſmall cannons, the firſt carrying balls but of 10 pounds, the other but of 5.

Note, For ſea-ſervice, they uſe cannons of different bores, that is, carrying balls of different weights, *viz.* 4, 6, 8, 12, 18, 24, and 36, pounders, becauſe culvarines would be too troubleſome upon the decks of ſhips. *

5. Cannons are again diſtinguiſhed, with reſpect to the richneſs or thickneſs of their metal, into three kinds, *viz.* the *Double fortified* the *Legitimate*, and the *Leſſened:* The double fortified have above nine times the diameter of its mouth in the outward circumference of its breech; the legitimate has preciſely nine times, and the leſſened leſs than nine times.

6. The carriage of a cannon ſerves to ſupport it; to turn it to the right and left, and to raiſe or depreſs it. The carriages uſed in ſhips are very fit for all theſe purpoſes, *Fig.* 2. as Fig. A, of which the wheels are

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* The Cannon us'd by the *Engliſh* in their Sea-ſervice, are of the following calibres, that is, carry balls of the following weights, *viz.* 1½, 3, 6, 9, 12, 18, and 24 pounds.

ſeldom more than 8 inches radius; B repreſents the hind wheels, C the fore wheels, D the bottom of the carriage, A the ſides or brackets, E the reſts for the trunnions, F the axletrees of the wheels, G the holes for the laſhing-faſt or ſecuring the piece. In the land-ſervice, they are obliged to mount their cannon upon carriages, better adapted for drawing them, having wheels of two or three foot radius, as Fig. H.

SUPPOSITIONS.

The general laws of motion are here ſuppoſed, the chief of which are laid down in the beginning of the tract relating to the working of a ſhip, and ſome others are here annexed.

1. A body in motion cannot communicate more of motion than is in itſelf.

2. There are but two ways that motion can be deſtroy'd; 1. When the moving body meets with an obſtacle it cannot overcome. 2. When it receives a new movement contrary to its former direction. When motion is deſtroy'd by an obſtacle it cannot overcome, it is intirely deſtroy'd at once; but when it is overcome by a contrary movement, it is only deſtroy'd in proportion to the quantity of ſuch contrary motion.

3. When one body in motion meets another, it communicates to it, if it can, a force neceſſary to put it in motion with an equal velocity with itſelf.

4. When one body in motion meets another body, it receives from it a contrary movement equal to that it communicates.

5. When a moving body has two contrary and equal motions, if it is intirely at liberty to move itſelf by their directions, the two ſeveral motions will deſtroy each other, and the body will remain immoveable; but if the body cannot obey the direction of the one, that motion will be deſtroy'd, and the other will remain intire.

6. If a moving body has two contrary and equal motions, and being at liberty to follow the direction of the one in its full velocity, is not at liberty to move according to the direction of the other, but with a part of its velocity, this ſecond motion will deſtroy a part of the other, in proportion to its own velocity, and will itſelf be intirely deſtroy'd.

Note, The foregoing laws of motion demonſtrate themſelves by continual experience.

PROP. I.

The Proportions of the ſeveral Parts of a Cannon.

The Proportion of the Parts of a Cannon depend upon ſo many circumſtances, that experience ſerves much better to determine them, than any ſtated rules: So that I ſhall content myſelf with giving the following Tables, wherein I have inſerted only the proportions of ſuch pieces, whoſe calibres are common both to the *French* and *Engliſh* uſage.

PRO-

Proportions *of Braſs Cannons.*

Calibre	Weight	Diameter of the outward Circuit.										Diamerer of the Bullet.		Length from the Caſcabell, to										Quantity of the Charge, for		
		at the mouth		at the Vent		1 Reinf.		2 Reinf.		under the Aſtragal				1. Reinf.		Trunnions		2. Reinf.		Aſtragal.		the muzle		Proof.	Salut.	Comb.
lb.	lb.	I.	L.	I.	L.	I.	L.	I.	L.	I.	L.	I.	L.	F.	I.	F.	I.	F.	I.	F.	I.	F.	I.	lb.	lb.	lb.
6	1600	3	6	10	7	9	11	8	10	6	2	3	5	2	4	3	4	3	6	6	8	7	0	6	$4\frac{1}{2}$	$3\frac{1}{2}$
12	3000	4	6	13	5	12	5	11	2	7	7	4	3	3	0	4	1	4	3	8	2	8	6	$9\frac{3}{4}$	$7\frac{1}{3}$	$5\frac{2}{3}$
18	4000	5	1	15	4	14	4	13	3	8	8	4	11	3	2	4	5	4	8	8	7	9	0	$12\frac{1}{3}$	$9\frac{1}{4}$	$7\frac{1}{4}$
24	5000	5	8	16	11	15	5	14	1	9	5	5	5	3	4	4	10	4	11	9	0	9	6	16	12	$9\frac{1}{3}$

N. B. I have included in theſe Tables only ſuch Pieces of Cannon, whoſe Calibres are common both to the *French* and *Engliſh* uſage.

PROPORTIONS *of Iron Guns.*

Calibre.	Diameter of the Chaſe.		Diameter of the Breech.			Diameter at the Trunnions.		Whole Length.	
Pounds.	Inches.	Lines.	Feet.	Inch.	Lin.	Inches.	Lines.	Feet.	Inches.
6	3	6	1	0	7	10	9	7	0
12	4	5	1	3	5	12	7	8	6
18	5	1	1	5	9	13	9	9	0

N. B. I have omitted the Table of the Proportions of Sea Carriages, ſince with us they are adjuſted by the Dimenſions of the Ports.

PROP.

Of the Cavity or Chaſe of a Cannon.

The Chaſe of a Cannon ought to be exactly in the middle, it ought to be equal throughout, and without creviſes or honey-combs.

1. Whether the chaſe be perfectly in the middle, may be tried by a cylinder of wood A B C, whoſe part A B is equal to the cavity of the cannon, and the part B C equal to its outward circumference. *Fig.* 3. Let the cylinder A B be put into the cannon, and applying a rule all round upon the lines B C parallel to the axis of the cylinder, you will ſee if it exactly agrees with the outſide of the cannon.

2. Whether the chaſe be equal throughout may be known, by applying rulers to its inward ſides; for if the rulers are parallel, 'tis a ſign the inward ſides of the cavity are ſo likewiſe, and conſequently that it is equal throughout. This equality of the cavity does not exclude the chamber, which may be made at the bottom of the cannon, as will be mention'd hereafter.

3. To know if a cannon has creviſes or honey-combs, after having fired it twice or three times with double charges, they uſe a ſort of rammer arm'd with flexible points, which cloſing towards the rammer in thruſting it into the cannon, extend themſelves, and entering into the creviſes, remain there upon a forcible drawing it back. *Fig.* 4. Another way of trying, is by applying a looking-glaſs A, to the mouth of the cannon, which *Fig.* 5.

will

will repreſent very plainly the oppoſite ſide B C, and ſucceſſively all the reſt, by changing the poſition of the cannon, and conſequently the creviſes, if there be any, will appear.

PROP. III.

Of the Outſide of a Cannon.

With reſpect to the outſide of a Cannon, regard muſt be had, 1. To the force of the powder and ball. 2. To the paralleliſm of the ſight and the chaſe. 3. To its beauty and conveniency of movement.

1. A cannon muſt be thickeſt towards the breech, becauſe the force of the powder is there greateſt, nevertheleſs its mouth muſt be alſo ſtrengthened with rings, leſt the bullet, in going out, ſhould ſplit the edges, not being of the ſame thickneſs of metal.

2. That the greater thickneſs of metal at the breech than at the mouth of the cannon, ſhould not hinder the ſight or aim, by preventing its paralleliſm with the chaſe or cavity, a button or nob ſhould be rais'd upon the upper part of the muzzle.

3. The other parts of the outſide of a cannon ſerve either to its ornament, as the carving, the devices, and inſcriptions ; or to its movement, as the dolphins and the trunnions.

LEMMA I.

Suppoſe the rectangular cylinder A B, whoſe baſes A B are circles ; I ſay that all the ſurfaces of the cylinder *Fig.* 6.

parallel to the baſe A, as C, will alſo be circles equal to the ſame baſe A. Imagine A B the axis of the cylinder, and the rectangle parallelogram A B D E, in turning about the axis A B, to have form'd the cylinder.

Demonſt. Since the ſurfaces A and C are parallel, the ſections C F, A E, will be alſo parallel (by 16. 11. of *Euc.*) and the figure A C F E will be a parallelogram, whoſe ſides A E, C F, will be equal; in the ſame manner it may be ſhewn, that the other radii of the circle A are equal to the radii of the circle C, which anſwer to them, and conſequently the circles A and C are equal.

LEMMA II.

In the rectangular cylinder E, ſuppoſe a circle parallel to the baſe E, having for its center B; if from the point A of the circumference of the ſaid circle B, be deſcrib'd a kind of a circle upon the concave ſurface of the cylinder, it will cut the circumference of the circle B in the point C, and the angles B A C, which the radii A C make with the radius A B, will be leſs than any other angle, as B A D, which another radius A D would form with the ſame radius A B. Suppoſe the line D F parallel to the axis of the cylinder, and perpendicular upon the plain of the circle B, then draw B F, B C, B D.

Fig. 7.

Demonſt. In the triangle B F D, the line B D oppoſite to the right angle B F D, will be greater (by 19. 1. *Euc.*) than B F, or B C; and ſince in the triangles B A C, B A D, the ſides B A, A D are equal to the ſides B A, A C, the baſe B D, which is greater than B C (by 18. 1. *Euc.*) will make the angle B A D greater than B A C.

Lbmma III.

Suppoſing, as in the foregoing, a plain to paſs through the point D, and the line A B; the line D D will be greater than C C.

Demonſt. In the triangles D A D, C A C, which have the ſides A D, A C equal, the angle D A D which is greater than C A C (by the prec.) will make the line D D greater than C C (by 19. 1. *Euc.*)

COROLLARY.

If the line C C is the leaſt of thoſe, which paſſing through the line A B, are determined by the circumference of the circle A, the triangle C A C is inſcribed in a circle equal to the baſe E of the rectangular cylinder.

Prop. IV.

Of the Calibre, or Bore of a Cannon.

1. The Calibre or Bore of a Cannnon is known by the diameter of its mouth, the cube of which, in compariſon with the diameter of a ball of one pound, gives the calibre of the cannon. Thus, when the diameter of the mouth of a cannon is double to that of one pound, the cannon will be an eight pounder; if triple, a 27 pounder; if quadruple, a 64 pounder, &c.

2. To find the calibre of the cannon B, open the compaſſes A as wide as they will ſtand in the mouth of the cannon B. *Fig.* 8.

3. But if it be required to find the calibre from the part A of a cannon; from ſome point A, deſcribe a kind of a circle, upon the cancave ſurface A, and taking the ſhorteſt diameter B C, by extending the compaſſes croſſways, as in the figure; then upon the line E F, equal to B C, form a triangle with ſides D E, D F, equal to the radii A B, A C; this triangle being inſcribed within a circle E D F G, will give D G the diameter of a circle equal to the mouth of the cannon, of which A is a piece, (by *Lem.* 3.) *Fig.* 9.

4. It would be convenient to have a rule, as A B, for meaſuring the calibre of cannon, of which the part A C being the diameter of a cannon of one pound, A D will be the diameter of one of 4, and A E of one of 8 pounds; ſo that by applying the diameter of a cannon to the rule, you will at once ſee its calibre. *Fig.* 10.

Note, The rule for meaſuring the calibre, is divided in the ſame manner as the rule for meaſuring ſolids on the Sector, or Compaſs of proportion, (*vide Appendix*)

PROP. V.

Of the Bullet.

A Bullet is commonly made of a ſolid globe of iron.

1. It is made of a ſpherical figure, as the moſt proper to receive, keep, and communicate motion.

2. It is made of iron, as being a metal of ſufficient hardneſs and weight, and of which there is a ſufficient plenty.

3. It is made ſolid, becauſe a hollow bullet would be liable to the ſame reſiſtance from the air, and the body it ſtrikes; but would want the force which ſolidity gives it.

PROP. VI.

Of the Calibre of the Bullet.

1. The Calibre of a Bullet is in the ſame manner known by its diameter taken between the points of a pair of Callipers, or curve-legg'd Compaſſes, the cube of which diameter, with reſpect to one of the weight of one pound, gives its calibre, and the proportion is the ſame in bullets as in cannons; only it ought to be obſerved, that the diameter of a bullet ought to be leſs by two lines, than the diameter of a cannon of the ſame calibre, that ſo it may enter eaſily.

2. On the other, hand we know the diameter of a bullet by its weight, or calibre; for the cube root of its weight gives exactly its diameter.

3. This ſhews a new way of dividing a rule for taking the calibre. For let A B repreſent a rule, of which the part A C is *Fig.* 10. the diameter of a bullet of one pound weight; let the whole rule be divided into 1000 equal parts, of which let A C be 10, then adding 3 cyphers to the weight of each bullet, extract the cube root, and you find the number of parts anſwering to the diameter of each bullet, almoſt to a thouſandth part. For example, if you would mark upon this rule for the calibre, that of a bullet of 24 pounds, adding 3 cyphers to 24, which makes 24000, the cube root thereof, *viz.* 29, gives the diameter of a bullet of 24 pounds. After which manner was compos'd the following table.

Weight of Bullets in Pounds.	1	2	4	6	8	12	18	24	36	48	64	100
Number of Parts in their Diameters.	10	13	16	18	20	23	26	29	33	36	40	46

4. To find the diameter of a bullet, of which we have only the convex part A; on the point A, as a center, diſcribe a circle upon the convex ſurface A, and having taken the diameter B C with a pair of callipers, then *Fig.* 11. upon the line E D equal B C, make a triangle, whoſe two ſides F E, F D let be equal to the lines A B, A C, this triangle cannot be

inſcribed but in the circle E F D G, the diameter of which F G, will be the diameter of the bullet.

Demonſt. The plain of the triangle A B C paſſing through the pole A, and through the diameter B C of a circle, will paſs through the center of a globe (by Prop. 1. *Sph. Trig.*) therefore the triangle ABC will be inſcribed in a circle, which will paſs through the center of the bullet, and conſequently will have for its diameter that of the bullet.

PROP. VII.

Of the Cauſe of the Motion of the Bullet in a Cannon.

Fig. 12. Let A B repreſent a Cannon, wherein the powder takes up the ſpace A, and the bullet the ſpace B; when the powder takes fire, it will fill a larger ſpace, as A C, and thruſt the bullet from B to C; from whence may be obſerved,

1. That the ſwiftneſs of the motion communicated to the bullet by the powder, ought to be meaſured by the line A C, the length of the ſpace gain'd by the powder in firing; becauſe in the time of the powder's firing, the bullet is forc'd with the velocity A C.

2. Whether the powder is inflam'd ſucceſſively, or all at once, the bullet receives an equal velocity, ſince it all takes fire before the bullet goes out of the cannon. For if at the firſt inſtant only half the powder ſhould take fire, inſtead of gaining the ſpace A C, it will only gain A D, and communicate to the bullet only *Fig.* 12.

only the velocity A D; but afterward as the reſt takes fire, it will communicate a ſecond velocity equal to D C.

Or it might have been thus.

Fig. 12. *Let* A E *repreſent a Cannon, wherein the powder takes up the ſpace* A, *and* B *is the bullet; the inſtant of the powder taking fire, the flame of it will with great force preſs the bullet forward, and fill a larger ſpace, as* A D, A C, *or even* A E *the whole bore; from whence may be obſerved.*

1. *That the ſwiftneſs of the motion given to the bullet is by the inſtantaneous expanſion of the flame of the fired powder, and that ſuch ſwiftneſs continually increaſes, whilſt the bullet is moving the whole length from* B *to* E, *if the expanſion of the flame is of force ſufficient to preſs out the bullet, if placed at reſt in* E.

2. *That the height of the charge if powder ſhould be proportioned to the diameter of the bore and length of the cannon, as a heavy bullet will require a greater force to give it a velocity equal to a lighter in proportion to their weight, and if the length of the cannon is increaſed, the expanſion of the flame muſt be increaſed, ſo as to preſs the bullet for the whole length, or elſe the length will diminiſh the carriage of the bullet.*

3. *That the increaſe of the weight of the bullet helps to the continuance of its velocity, as it has thereby the more force to overcome the obſtacles it meets with.*

4. *That the increaſe of the weight of the bullet diminiſhes the expanſion of the flame of the powder; for though the bullet has not any contrary motion, yet as it muſt receive its whole motion from ſuch expanſion, it will, in reſiſtance thereto, communicate a contrary motion to that it receives, and ſuch contrary motion increaſes in proportion to the weight of the bullet.*

COROLLARIES.

Fig. 13. 1. If two equal bullets, A and B, are thrown out of equal cannons, their velocities will be in the ſame proportions as the quantity of powder in the one and the other cannon, ſince the ſpaces A C and B D will be in the ſame proportion as the ſpaces A and B.

Fig. 14. 2. If the bullets A and B, of different calibres, be thrown out with proportionable charges, their continued velocities will be as the cube roots of their calibres, or as their diameters, ſince the lines A C and B D increaſe in the ſame proportion as the cube roots of the quantities of the charges, or of the bullets.

3. It will be eaſy to determine the proportion of velocity betwixt bullets of different calibres, whoſe ſeveral charges are determined. For 1. If the charges are proportional, their continued velocities will be as their diameters. 2. In diminiſhing or augmenting the charges, their velocities are dimiſhed or augmented in the ſame proportion.

Mr. Robins, *in his new principles of Gunnery (hereafter mention'd) proves that all the powder of the charge* *is fired, before the bullet is ſenſibly moved from its place; and after determining the force of our government gun-powder, gives rules (that have relation to the length and calibre of the cannon and bullet) for finding the velocities given by different charges to all kinds of bullets, and the charges that will give the greateſt velocity to the different kinds of cannon and bullets.*

PROP. VIII.

Of the Chamber, where the Powder is plac'd in a Cannon.

Fig. 15. 1. The Chamber A ſerves to inflame a greater part of the powder; for the powder A taking fire in the middle, is forc'd againſt the bounds of the chamber, from whence it is reverberated againſt the fire.

2. Nevertheleſs, chambers are ſeldom made in cannons, becauſe they would thereby be rendered more difficult to be charged and diſcharged, and after firing oftentimes, ſome of the fire would remain, which would be dangerous to the Cannoniers.

PROP. IX.

Of the length of a Cannon.

1. It is certain, that the length of a cannon diminiſhes the motion of the ball, by its rubbing againſt the ſides in paſſing.

2. The length of a cannon ſerves to augment the motion of the bullet, becauſe it gives time to the powder to take fire in a greater quantity before the ball gets out.

3. Experience only can determine the length of a cannon, becauſe its effect depends upon certain circumſtances, which cannot be exactly known; as the quality of the powder, the reſiſtance the bullet finds in rubbing, &c.

PROP. X.

Of the charging a Cannon.

1. Tho' experience muſt regulate the quantity of powder, yet in general it may be obſerved, that in proving a cannon, the charge may be much greater than in battle, or when the cannon grows hot.

2. The bullet muſt enter eaſily, leſt it damage the metal in going out, or, it may be, burſt the cannon. Care ſhould therefore be likewiſe taken, that there be no inequalities in the ball, at leaſt that it will enter eaſily every way.

3. It would be eaſy to burſt a cannon, if after having put in the bullet B, the wedge C be inſinuated of a ſufficient hardneſs to reſiſt the force of the bullet, for then the bullet cannot paſs without burſting the cannon.

PROP. XI.

Of a Cannon's heating.

1. When a cannon grows hot, it recoils with greater violence upon the carriage, becauſe the powder takes fire quicker, and in a greater quantity.

2. Although the powder ſtrikes equally againſt both the upper and lower parts of the cannon, yet notwithſtanding it will recoil; ſince it is not at liberty to move downwards; for the lower part being forc'd againſt the carriage, receives from that reſiſtance a contrary motion (by *Sup.* 4.)

3. The heat of the cannon commonly increaſes the velocity of the bullet; for that it cauſes a greater quantity of powder to take fire before the bullet goes out of the cannon; yet it may happen to ſhorten the carriage of the ball, if the cannon recoiling, falls again before the bullet gets out; for that motion of the fall of the cannon being communicated to the bullet, very much diminiſhes the length of its carriage, as ſhall be ſhewn farther on.

PROP. XII.

Of the horizontal Carriage of a cannon.

Let the cannon A B be diſpoſed horizontally, the bullet B will not keep the horizontal line A B C, but a curve line, which will carry it to the point D below the point C. *Fig.* 16.

Demonſt. The horizontal motion impreſs'd on the bullet by the powder, not being contrary to the motion of its own gravity, will not deſtroy it; therefore the bullet will be carried downwards, being moved by a compounded motion, partly horizontal, and partly vertical, which will conſequently carry it to the point D below the point C.

COROLLARY.

From whence it follows, that when a cannon ſeems to carry point-blank, or to raiſe its ſtroke, the line of aim is not parallel to the cavity of the cannon, for then its carriage would be too low.

REMARKS.

1. It is not neceſſary to examine the nature of the line deſcrib'd by the bullet in its paſſage, it being demonſtrable, that it would be parabolical, if the reſiſtance of the air, or the directions of its own gravity, did not affect it. It is ſufficient to know, that the line B D, the paſſage of the bullet, is not the right line B F D. Let the right line B F D be therefore equally divided in the point F, and draw F E parallel to C D, and F G parallel to B C.

Demonſt. The line E F being equal to the line G C, the bullet will take up the ſame time in paſſing from the point B to the line E F, by reaſon of its horizontal motion, as in paſſing from the line E F to the line C D; therefore when it comes to the line E F, it will have deſcended but the fourth part of the line C D (by *Prop.* 10. *Mechan.*) and not its half part E F. Therefore the paſſage of the bullet will be above the point F.

2. The greater the velocity of the bullet is, the leſs will be its deſcent at the ſame diſtance; becauſe it takes up a leſs time in its paſſage, and the quantity of time determines the ſpace or quantity of its deſcent. So that the bullet which was carried from B to D, would have been carried from B to H, if it had been thrown by the cannon with a double velocity; becauſe it would then have taken up but half the time in paſſing from the point B to the line C D.

3. If the curve line of the bullet's paſſage did not bring it to the ground, it would not ſtop till the reſiſtance of the air intirely deſtroy'd its horizontal motion, and its progreſs would conſequently become immenſe.

4. Horizontal cariages are to one another, according to their ſeveral velocities; if no regard be had to the reſiſtance of the air.

Demonſt. If in the ſame time that one bullet paſſes, by its horizontal motion, from the point A to the line C E, another bullet is only carried from the point A to the line B D; if further, the firſt bullet in its paſſage from A towards C, falls upon the ground at the point E, the other bullet falls equally, and ſtrikes the earth at the point D; conſequently as A C the meaſure of the velocity of the firſt bullet, will be the meaſure of its carriage; ſo A B the meaſure

Fig. 17.

meaſure of the velocity of the ſecond bullet, will alſo be the meaſure of its carriage.

PROP. XIII.

Of the reſiſtance of the Air.

1. The reſiſtance of the air is nothing elſe, but the motion impreſs'd upon the bullet by the particles of the air repelling it.

2. The reſiſtance of the air increaſes in a duplicate ratio to the velocity of the bullet; for one bullet having twice the velocity of another, ſtrikes againſt a double quantity of air in the ſame time, and likewiſe with a double force.

3. The reſiſtance of the air diminiſhes the carriage of a bullet in a duplicate ratio to its velocity; becauſe it diminiſhes its carriage in the ſame proportion as it diminiſhes its motion, or as it impreſſes or increaſes a contrary motion.

Or it might have been thus.

4. *The reſiſtance of the air to the carriage of different bullets, is in proportion to the ſquares of their diameters, but not to their maſs or quantities of mattter; therefore their velocities, when given equal, will be unequally diminiſhed.*

Demonſt.

Fig. 18. *Demonſt.* The bullets A and B having the ſame velocity, will find the reſiſtance of the air in proportion to the areas of the ſpace each paſs through, but not to their maſs or quantity of matter; their motion will then not be diminiſhed in the ſame ratio as their maſs; therefore their velocities or their carriage will be unequally diminiſhed.

COROLLARY.

Though the carriage of one cannon be given with all its circumſtances, we cannot from thence determine the carriage of another exactly, without regard to the reſiſtance of the air, which cannot certainly be known.

5. The thicker the air, the greater the reſiſtance, and the cariage of the bullet will be diminiſhed in proportion to ſuch denſity, ſince the bullet muſt conſequently ſtrike againſt a greater number of particles.

6. If two bullets paſs through different airs with different velocities, the reſiſtances they meet with will not be equal, though ſuppoſing the denſities and velocities are in reciprocal proportion; but the bullet which paſſes through the ſpace of thick air, will find leſs reſiſtance in the ſame proportion as their velocities.

Demonſt. If one of the bullets paſs through air twice as thick as the other, and the other with double velocity to the firſt, they will then both be ſtruck with an equal number of particles of air; but the ſecond will be ſtruck with a double force.

COROLLARY.

If the bullets A and B, equal in every reſpect, are thrown with equal force, *Fig.* 19. and the bullet A paſſes through half its courſe in a thick air, and the reſt through a more refin'd air; if on the contrary, the bullet B paſs at firſt through a finer air, and continues the reſt of its courſe through a thick air; theſe two bullets will find equal reſiſtance, and will be equal in their carriage, ſuppoſing the air A twice as thick as the air B.

Demonſt. The bullet A being come to the middle of the thick air, will be ſo much overcome by the reſiſtance it meets with, and will have loſt ſo much of its velocity as the bullet B, at the end of the thin air; ſince it will have ſtruck againſt an equal number of particles, and with an equal force. Again, they will find an equal reſiſtance in the remainder of their courſe, the bullet A in paſſing through the remaining half of the thick air, and the bullet B in paſſing through the firſt half of the thick air. Further, the bullet A will find the ſame reſiſtance in paſſing through the thin air B, as the bullet B in paſſing through the remainder of the thick air A.

The ingenious Mr. Benj. Robins *F. R. S. has lately publiſhed new principles of gunnery, wherein he determines the force of gun-powder, and inveſtigates the difference of the reſiſting power of the air to ſwift and ſlow motions; and ſhews by ſeveral experiments,* *that its reſiſtance to ſwift motions is vaſtly greater than according to the foregoing doctrine, which, he ſays, holds true only in ſlow motions; and that if the bullet has the velocity given to it equal to the motion of* 1700 *feet in one ſecond of time (which he found by ſeveral very curious experiments made by the bullet impinging into a pendulum, to be nearly the greateſt velocity given to a muſket or a cannon ball) the reſiſtance of the air is greater than in the doctrine of ſlow motions nearly in the ratio of* 3 *to* 1; *and that if the flight of a muſket-ball of* $\frac{3}{4}$ *of an inch diameter, fired with half its weight of powder, from a piece* 45 *inches long, with a velocity equal to* 1700 *feet in* 1″, *was in the curve of a parabola, its horizontal range at* 45° *elevation, would be about* 17 *miles; whereas all agree that this range is really ſhort of half a mile; therefore the track deſcribed by ſuch a bullet greatly differs from a parabola, as it flies not the* $\frac{1}{34}$ *part of what it ought to do, if moving in that curve; and this contraction will not be wonder'd at, when it ſhall be conſider'd, that the reſiſtance the air gives to this bullet at its firſt iſſuing from the piece, is about* 120 *times the weight of the bullet.*

He alſo proves that a cannon-ball of 24℔ *fired with* 16℔ *of powder (which is the uſual battering charge) acquires the velocity of about* 1650 *feet in a ſecond, and the air's reſiſtance to this ball, at its firſt iſſuing from the piece, is more than* 20 *times its weight, and that if the flight of this ball was in the curve of a parabola, its greateſt horizontal range would be about* 16 *miles, which is between* 5 *and* 6 *times its real quantity, as it will always fall ſhort of* 3 *miles.*

PROP.

PROP. XIV.

Of the reſiſtance of Water, and the reboundings of the Bullet therein.

Water makes a greater reſiſtance than air, both becauſe it is a more denſe body, and that its parts are more difficult to divide; therefore bullets meet with ſo great reſiſtance in water, that they ſometimes rebound, and make, as it were, repeated leaps upon the ſurface of the water, as in the figure.

Fig. 20. Suppoſe the bullet A, which is carried againſt the ſurface of the water B, along the curve line A B inclined upon the ſame ſurface. 1. If the bullet was without weight, and the water ſolid at the point B, it would be forc'd back through the line B C, equal in every reſpect to the line B A, ſince its motion parallel to the water would remain the ſame, and the water would communicate to it a motion repelling it, equal to that with which it approached (by *Sup.* 4.)

2. The water not being ſolid, the air preceding the bullet, will make, as it were, a cavity in it in ſuch manner, that the bullet inſtead of ſtriking it at the point B, will not meet with it *Fig.* 21. but at the point D; or it will enter into the water by ſome curve line as D G, if it has force enough to divide it at the point D, as if it were thrown againſt the water by a line leſs inclined.

3. But if the bullet is carried with ſo much velocity, and ſo obliquely againſt the water, that it cannot divide it, it then receives a motion repelling it from the water with a force equal to that with which it approach'd it; and in ſuch caſe, if the bullet was without weight, it would be repell'd by the line D C equal in every reſpect to the line D B A.

4. The weight of the bullet cauſes the line D E, through which it is repell'd, to be much more inclined than the line D C; ſo that the bullet riſes out of the water at the point F, after having paſs'd under the water through the whole ſpace B D F.

LEMMA I.

Fig. 22.

If from the end A of the diameter A H, be drawn the lines A C, A E, extended to the circumference of the circle, and from the points C E be drawn the lines C D, E G, perpendicular to the diameter; the line A D will be to the line A G, as the ſquare of the line A C to the ſquare of the line A E.

Demonſt. The ſquare of A C is equal to the ſquares of A D and D C (by 47. 1. *Euc.*) and ſince the ſquare of D C is equal to the rectangle A D H (by 32. 3. *Euc.*) the ſquare of A C will be equal to the ſquare of A D, and the rectangle A D H; or it will be equal to the rectangle H A D: In the ſame manner the ſquare of A E will be equal to the rectangle H A G: But as the rectangle H A D: is to the rectangle H A G : : A D : A G (by 1. 6. *Euc.*) therefore as the ſquare of A C : ſquare A D : : A D : A G.

COROLLARY.

Since (by *Prop.* 10. *Mechan.*) the deſcents of heavy bodies are to one another, as the ſquares of their times; if the times be meaſured by the lines A C, A E; the deſcents muſt be meaſured by the lines A D, A G. Thus, ſuppoſing that in the time meaſured by A C, a heavy body in its deſcent would paſs through the line A D; then would it run through the line A G in the time meaſured by A E.

LEMMA II.

Fig. 23. If the body A receives at the point A a motion upwards, which motion being inſenſibly deſtroy'd by its gravity, ceaſes at the point B; it will be juſt the ſame time in its deſcent from B to A, as it took up in its aſcent from A to B; becauſe its motion of gravity will begin and increaſe by the ſame degrees, as its former motion decreaſed and ended.

COROLLARY.

Suppoſe the line A C elevated above the horizontal line A B, and the bullet A thrown to the ſame height, with a motion which would carry it to the point C in a minute of time, if the ball was without weight; but that by reaſon of its gaavity it deſcends again to the point B in the vertical line C B; we may thence *Fig.* 22.

thence deduce the following conſequences: 1. The ball will arrive at the point B in the ſame time it would be in coming to the point C; ſince its gravity makes no alteration in its horizontal motion, which carries it from the vertical line A H, to the vertical line B C. 2. The ball will take up one half of the ſaid time in its aſcent, and the other half in its fall. (by *Lem.* 2.) 3. The ball will aſcend but the fourth part of the line B C, or half that height it would have aſcended, in half the ſaid minute of time (by *Prop.* 10. *Mechan.*) if the motion cauſing its aſcent had continued always equal, and its gravity had not inſenſibly overcome it.

PROP. XV.

Suppoſe from the mouth of a cannon A, whoſe elevation is A C, a ball being thrown ſhould fall upon the point B of the horizontal line A B. Draw A H and B C perpendiculars upon the line A B, and from ſome point as D, deſcribe the circle A C H. I ſay, that if the cannon be ſet to any other elevation as A E, every other particular remaining the ſame, the line E F perpendicular to A B, will determine the point F, where the ball will fall.

Fig. 22.

Demonſt. Since the ball, at the elevation A C, falls upon the point B, it muſt needs be that during half the time it would take up in paſſing through the line A C, if its gravity did not affect it, its weight will cauſe it to deſcend the fourth part of the line B C (by *Cor. prec.*) But if during the time meaſured by half the line A C, the ball

deſcends

deſcends one fourth part of the line B C or D A; then during the time meaſured by half the line A E, it will alſo deſcend one fourth part of the line A G or E F; (by *Lem.* 1.) therefore the point F will be the place where the ball will fall at the elevation A E.

PROP. XVI.

Fig. 24. Suppoſe the line A K rais'd above the horizon A B, and the ball A having the elevation A C, to fall upon the point K: Draw A N perpendicular to A B, and K C parallel to A N. Again, let A H be drawn perpendicular to A K, and from ſome point of the line A H, deſcribe a circle A C H. I ſay, that if to the ball A be given any other elevation as A E, the line E F parallel to the line A N, will determine the point F, where the ball will fall upon the line A K. Draw C D and E G, perpendiculars upon A H, which will cut A N in the points M L.

Demonſt. The ſquare of A C is to the ſquare of A E; as A D to A G (by *Lem.* 1.) or as A M to A L (by 2. 6. *Euc.*) or as C K to E F. Therefore (by the prec.) if the ball A having the elevation A C fall upon the point K, it will fall upon the point F, if it obtains the elevation A E.

REMARK.

If the carriage and elevation of a cannon be given, together with the plane upon which the ball is to fall; we have all others upon every kind of plane; becauſe the force of a cannon may always be meaſured by the vertical line A N, which ought to be the ſame whether the plane be horizontal or not, ſince the plane has no effect upon the vertical carriage, by which the ball is returned towards the point A.

COROLLARY.

If the line K C be a tangent to the circle, the line A K is the greateſt carriage, and B A C the angle of elevation equal to 45 degrees, with half the angle of B A K; for the angle C A K is equal to half the angle N A K, which is 45 degrees, leſs half H A N, or B A K.

From whence is deduced this general rule:

To have the greateſt carriage upon a plane elevated above the horizon, to the elevation of 45 degrees, muſt be added half the elavation of the plane.

PROP. XVII.

The ſame reaſoning holds good, when the line A K lies inclined below the horizontal line A B, after having deſcrib'd the ſemicircle A C H, touching the line A K. *Fig.* 25.

COROLLARY.

In the ſame manner, *To have the greateſt carriage* A K, *deduct from the elevation of* 45 *degrees half the angle* B A K.

PROP. XVIII.

Of the Inſtruments uſed to find the elevation of a Cannon.

1. Having a ſquare of wood, or of *Fig.* 26. metal, as A B, one of whoſe ſides A C, let be prolong'd at diſcretion to D. From the point A, deſcribe the quadrant C E, in ſuch manner that A B may be perpendicular to A D; then divide the quadrant E C as uſual, beginning from the point E, and let a plummet be hung upon the point A: Laſtly, let the whole ſquare A B be filled up with lines parallel to A D, and equidiſtant from one another.

2. To uſe this inſtrument, put the *Fig.* 27. ſide C D into the cannon, applying it cloſe to the under ſide of the cavity or chaſe, ſo ſhall the plumb line A F ſhew upon the arch E C, the angle F A E, equal to the angle A B F, the elevation of the cannon above the horizontal line B F.

3. Beſides the above, there may be *Fig.* 28. provided a ſquare of paſte-board, or of metal, whoſe ſides G H, G I muſt be equal to the ſides A C, A E of the other inſtrument; then deſcribing upon G H a ſemi-circle, which ſpace muſt be cut away and left open, and divide the ſide G I into ſeveral equal parts.

4. Its uſe is as follows; when you would determine the horizontal carriage, place the point G of the paſte-board upon the point A of the other Inſtrument, and the line G I, upon the ſide A E; ſo ſhall one of the parallels to the line A D, touching the open ſemicircle in the point K, ſhew upon the ſcale G I, the greateſt horizonial carriage at the elevation H K; the other parallels will likewiſe ſhew the other carriages upon the ſcale G I, with their correſpondent elevations. Having therefore fix'd upon a certain carriage, put the inſtrument into the cannon, and raiſe the piece till the plumb line cuts upon the ſemicircle G K H, the correſpondent elevation.

Fig. 29.

5. When we deſire to know carriages elevated above the horizontal, we place the ſide G I of the paſte-board under the line A E, ſo that the angle E G I be the angle of the carriage with the horizontal. If on the other hand, we would know a carriage inclined below the horizon, we elevate the line G I above the line A E, to the ſame angle. All the reſt is done as above.

Fig. 30.

Fig. 31.

REMARK.

Carriages may be determined, not only by the greateſt, as has been above laid down, but even by any other, whoſe length and elevation is known, ſince it only depends upon determining the value of the ſcale G I.

PROP. XIX.

Of the force of the Ball againſt the Body it ſtrikes.

1. It is plain that a ball ſtriking a body ſoon after it paſſes out of the mouth of a cannon (every thing elſe being equal) will ſtrike it with greater force than after its motion is diminiſhed by the reſiſtance of the air.

2. If the ball A, at its paſſing out of *Fig.* 32. the mouth of a cannon, immediately meets with the body B, it may happen, that the air forc'd by the flame coming out of the cannon, and compreſs'd againſt the body B, may repel the ball with a force that may more leſſen its motion than the air it would meet with at a little greater diſtance; and in that caſe the ball will ſtrike the body B with leſs force, than it would ſtrike the body C at a little greater diſtance, beyond the reach of the flame of the powder.

3. If the ball A is carried againſt *Fig.* 33. the body B, in the perpendicular line A B, it will communicate its whole motion; but if the ball paſſes through the oblique line A C, the motion it communicates will be only in the ratio of the line A C to the line A D.

COROLLARY.

From whence it follows, that balls ſtrike with more force againſt rough ſtones, than againſt ſmooth ones.

PROP. XX.

Of Mortars for Bombardments.

Fig 34.

Mortars are a ſort of cannon, more ſhort and large than thoſe before deſcribed; their uſe is to throw bombs, and carcaſes; however, all that has been ſaid of other cannons may be applied to theſe, and particularly the rules given for determining their carriages and elevations; what more peculiarly relates to them may be comprehended in the following obſervations:

1. Mortars ought to be very ſhort, that the bomb may be plac'd in a proper ſituation, that neither the fuſe may be loſt in rolling through the cannon, nor the bomb burſt in its paſſage, nor even retarded by rubbing againſt the ſides of the cannon.

2. Mortars commonly have chambers, that the powder may the ſooner take fire in a greater quantity, there being no danger that fire ſhould remain in them unperceiv'd.

3. Some mortars, as A, are ſo contriv'd, as to be rais'd or depreſs'd upon their carriages, for the better regulating the diſtance of their carriage; but this ſort of mortars are more liable to be diſmounted, and more difficult to be fix'd in battery.

4. Mortars which, as B, are caſt with their carriage, are generally more eſteemed. They are commonly made to the elevation of 45 degrees,

becauſe that elevation gives the greateſt horizontal carriage. Nevertheleſs, as mortars are often deſigned to throw bombs into places elevated above the plane they are fix'd in, I would adviſe the giving them a greater elevation upon their carriages: And if there be occaſion to leſſen the carriage of ſuch mortars, it may be done by diminiſhing the quantity of powder in their charging; which ſeems more natural than leſſening the carriage, by giving the mortar an elevation leſs proper to throw the bomb.

FINIS.